8.38 (a) The mass moves down distance 1.20 m + x. Choose $y = 0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}\,kx^2$$

$$(1.50\text{ kg})9.80\text{ m/s}^2\,(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})\,x^2$$

$$0 = (160\text{ N/m})x^2 - (14.7\text{ N})x - 17.6\text{ J}$$

$$x = \frac{14.7\text{ N} \pm \sqrt{(-14.7\text{ N})^2 - 4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x = \frac{14.7\text{ N} \pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the mass will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})1.63\text{ m/s}^2\,(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})\,x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143\text{ m}}$

(c) The full work-energy theorem has one more term:

$$mgy_i + fy_i \cos 180° = \frac{1}{2}\,kx^2$$

$$(1.50\text{ kg})\,9.80\text{ m/s}^2\,(1.20\text{ m} + x) - 0.700\text{ N}(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})\,x^2$$

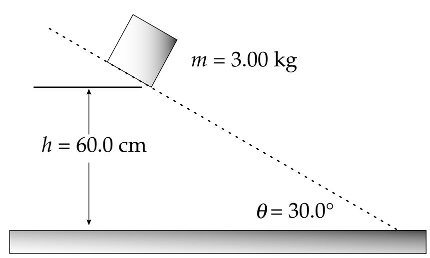
$$17.6\text{ J} + 14.7\text{ N}\,x - 0.840\text{ J} - 0.700\text{ N}\,x = 160\text{ N/m}\,x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371\text{ m}}$$

8.39 Choose $U_g = 0$ at the level of the horizontal surface.



Then $\Delta E = (K_f - K_i) + (U_{gf} - U_{gi})$ becomes:

$$-f_1 s - f_2 x = (0 - 0) + (0 - mgh)$$

or $\quad -(\mu_k mg\cos 30.0°)\left(\frac{h}{\sin 30.0°}\right) - (\mu_k\,mg)x = -mgh$

Thus, the distance the block slides across the horizontal surface before stopping is:

$$x = \frac{h}{\mu_k} - h\cot 30.0° = h\left(\frac{1}{\mu_k} - \cot 30.0°\right) = (0.600\text{ m})\left(\frac{1}{0.200} - \cot 30.0°\right)$$

or $\quad x = \boxed{1.96\text{ m}}$

***8.40** The total mechanical energy of the diver is $E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh$. Since the diver has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv$$

The rate he is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}$$

8.41 $U(r) = \frac{A}{r}$

$$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$$

8.42 $F_x = -\frac{fU}{fx} = -\frac{f(3x^3y - 7x}{fx} = -(9x^2y - 7) = 7 - 9x^2y$

$$F_y = -\frac{fU}{fy} = -\frac{f(3x^3y - 7x}{fy} = -(3x^3 - 0) = -3x^3$$

Thus, the force acting at the point (x, y) is

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} = \boxed{(7 - 9x^2y)\mathbf{i} - 3x^3\,\mathbf{j}}$$

***8.43** (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

> At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
> At $r = 2.3$ mm, the equilibrium is unstable.
> A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The particle energy cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\text{ J} \le E < 1\text{ J}}$.

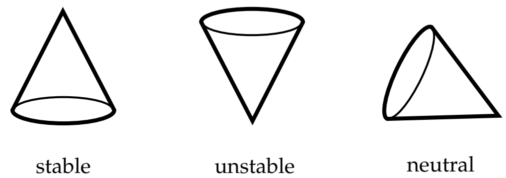
(c) If the particle energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, its position is limited to $\boxed{0.6\text{ mm} \le r \le 3.6\text{ mm}}$.

(d) $K + U = E$. Thus, $K_{\max} = E - U_{\min} = -3.0\text{ J} - (-5.6\text{ J}) = \boxed{2.6\text{ J}}$

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\text{ mm}}$.

(f) $-3\text{ J} + W = 1\text{ J}$. Hence, the binding energy is $W = \boxed{4\text{ J}}$.

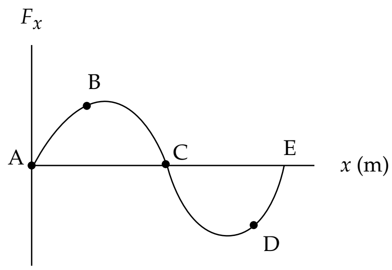
***8.44**



8.45 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

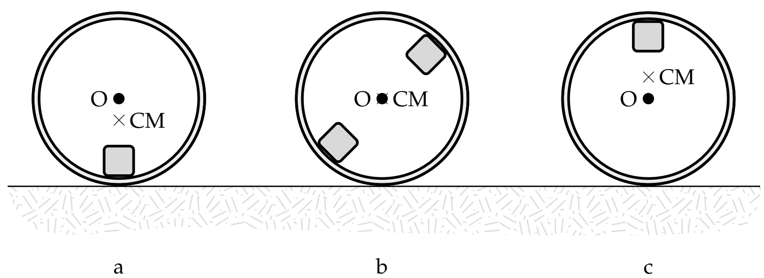
(c)



8.46 (a) As the pipe is rotated, the CM rises, so this is *stable* equilibrium.

(b) As the pipe is rotated, the CM moves horizontally, so this is *neutral* equilibrium.

(c) As the pipe is rotated, the CM falls, so this is *unstable* equilibrium.



8.47 (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2 + L^2}$, so each exerts force $k(\sqrt{x^2 + L^2} - L)$ toward its fixed end. The y-components cancel out and the x-components add to:

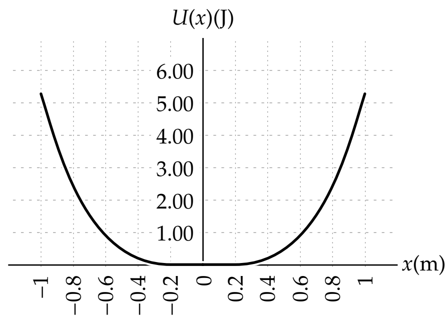
$$F_x = -2k(\sqrt{x^2 + L^2} - L)\left(\frac{x}{\sqrt{x^2 + L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}\right) dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2 + L^2}} dx$$

$$U(x) = \boxed{kx^2 + 2kL(L - \sqrt{x^2 + L^2})}$$

(b) $U(x) = 40.0x^2 + 96.0(1.20 - \sqrt{x^2 + 1.44})$



x, m	0	0.200	0.400	0.600	0.800	1.00	1.50	2.00	2.50
U, J	0	0.011	0.168	0.802	2.35	5.24	20.8	51.3	99.0

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

(c) $K_i + U_i + \Delta E = K_f + U_f$

$$0 + 0.400 \text{ J} + 0 = \frac{1}{2} m v_f^2 + 0$$

$$v_f = \boxed{\sqrt{\frac{0.800 \text{ J}}{m}}}$$

8.48 (a) $E = mc^2 = (9.11 \chi 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = \boxed{8.19 \times 10^{-14} \text{J}}$

(b) $\boxed{3.60 \times 10^{-8} \text{J}}$

(c) $\boxed{1.80 \times 10^{14} \text{J}}$

(d) $\boxed{5.38 \times 10^{41} \text{J}}$

8.49 (a) Rest energy $= mc^2 = (1.673 \infty 10^{-27} \text{ kg})(2.998 \infty 10^8 \text{ m/s})^2 = \boxed{1.50 \times 10^{-10} \text{J}}$

(b) $$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - (v/c)^2}} = \frac{1.50 \times 10^{-10} \text{ J}}{\sqrt{1 - (.990)^2}} = \boxed{1.07 \times 10^{-9} \text{J}}$$

(c) $K = \gamma mc^2 - mc^2 = 1.07 \times 10^{-9} \text{ J} - 1.50 \times 10^{-10} \text{ J} = \boxed{9.15 \times 10^{-10} \text{J}}$

8.50 The potential energy of the block is mgh.

An amount of energy $\mu_k mgs \cos\theta$ is lost to friction on the incline.

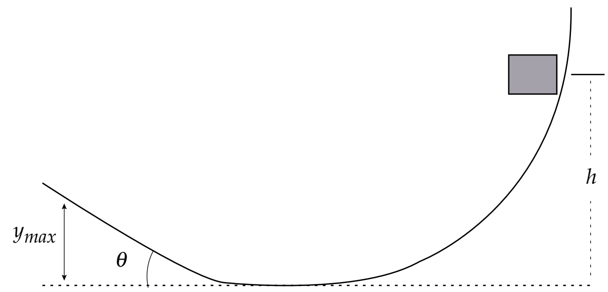
Therefore the final height y_{max} is found from

$$mgy_{max} = mgh - \mu_k mgs \cos\theta$$

where

$$s = \frac{y_{max}}{\sin\theta}$$

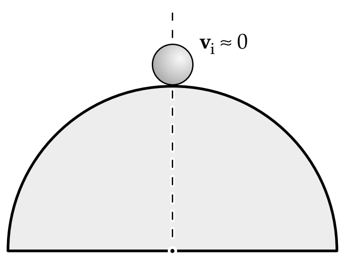
$$\therefore\ mgy_{max} = mgh - \mu_k mgy_{max} \cot\theta$$

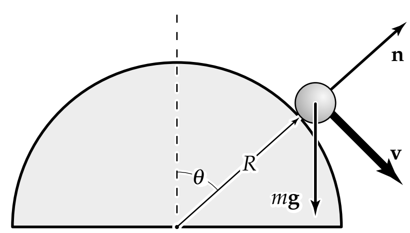


Solving,

$$y_{max} = \frac{h}{1 + \mu_k \cot\theta}$$

***8.51** m = mass of pumpkin
R = radius of silo top





$$\Sigma F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin is ready to lose contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

Choose $U_g = 0$ in the $\theta = 90.0°$plane. Then applying conservation of energy from the starting point to the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

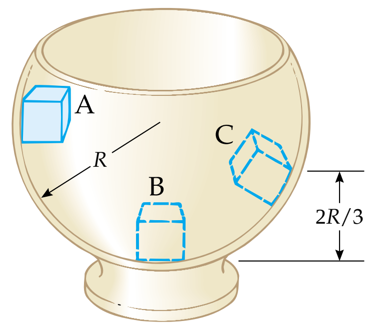
Using the result from the force analysis, this becomes

$$\frac{1}{2}\, mRg \cos\theta + mgR\cos\theta = mgR, \text{ which reduces to}$$

$$\cos\theta = \frac{2}{3}\text{ , and gives } \theta = \cos^{-1}(2/3) = \boxed{48.2ϒ}$$

as the angle at which the pumpkin will lose contact with the surface.

8.52 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$



(b) $K_A + U_A = K_B + U_B$

$$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$$

(c) $$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$$K_C = K_A + U_A - U_C = mg(h_A - h_C)$$

$$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$$

8.53 (a) $$K_B = \frac{1}{2}\, mv_B^2 = \frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$$

(b) $\Delta E = \Delta K + \Delta U = K_B - K_A + U_B - U_A$

$$= K_B + mg(h_B - h_A)$$

$$= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m})$$

$$= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}$$

(c) It's possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

***8.54** $v = 100\text{ km/h} = 27.8\text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}\, D\rho A v^2 = \frac{1}{2}(0.330)(1.20\text{ kg/m}^3)(2.50\text{ m}^2)(27.8\text{ m/s})^2 = 382\text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$P = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv \sin\theta$$

$$P = (382\text{ N})(27.8\text{ m/s}) + (1500\text{ kg})(9.80\text{ m/s}^2)(27.8\text{ m/s})\sin 3.20°$$

$$P = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

***8.55** At a pace I could keep up for a half-hour exercise period, I climb two stories up, forty steps each 18 cm high, in 20 s. My output work becomes my final gravitational energy,

$$mgy = 85\text{ kg}(9.80\text{ m/s}^2)(40 \times 0.18\text{ m}) = 6000\text{ J}$$

making my sustainable power

$$\frac{6000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$$

8.56 $k = 2.50 \times 10^4$ N/m $\quad m = 25.0$ kg $\quad x_A = -0.100$ m $\quad U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $$E = K_A + U_{gA} + U_{sA} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E = (25.0\text{ kg})(9.80\text{ m/s}^2)(-0.100\text{ m}) + \frac{1}{2}(2.50 \leftrightarrow 10^4\text{ N}/\mu)(0.100\mu)^2$$

$$E = -24.5\text{ J} + 125\text{ J} = \boxed{100\text{ J}}$$

(b) Since only conservative forces are involved, the total energy at point C is the same as that at point A.

$$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$0 + (25.0\text{ kg})(9.80\text{ m/s}^2)x_C + 0 = 0 + -24.5\text{ J} + 125\text{ J} \Rightarrow x_C = \boxed{0.410\text{ J}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$

$$\frac{1}{2}(25.0\text{ kg})\,v_B^2 + 0 + 0 = 0 + -24.5\text{ J} + 125\text{ J} \Rightarrow v_B = \boxed{2.84\text{ m/s}}$$

(d) K and v are at a maximum when $a = \dfrac{\Sigma F}{m} = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force). This occurs at $x < 0$ where

$$k|x| = mg \quad \text{or} \quad |x| = \frac{(25.0 kg)(9.80 m/s^2)}{2.50 \times 10^4\text{ N}/m} = 9.80 \times 10^{-3}\text{ m}$$

Thus, $K = K_{max}$ at $x = \boxed{-9.80\text{ mm}}$

(e) $K_{max} = K_A + (U_{gA} - U_g|_{x=-9.80\text{ mm}}) + (U_{sA} - U_s|_{x=-9.80\text{ mm}})$, or

$$\frac{1}{2}(25.0\text{ kg})\,v_{max}^2 = (25.0\text{ kg})(9.80\text{ m/s}^2)[(-0.100\text{ m}) - (-0.0098\text{ m})]$$

$$+\frac{1}{2}(2.50 \times 10^4\text{ N/m})\,[(-0.100\text{ m})^2 - (-0.0098\text{ m})^2]$$

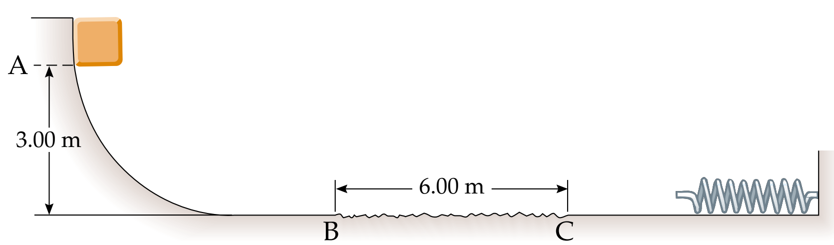
yielding $v_{max} = \boxed{2.85\text{ m/s}}$

8.57 $\Delta E = W_f$

$E_f - E_i = -f \cdot d_{BC}$

$\frac{1}{2}\,k\,\Delta x^2 - mgh = -\,\mu mgd$

$$\mu = \frac{mgh - \frac{1}{2}k \cdot \Delta x^2}{mgd} = \boxed{0.328}$$



Goal Solution

G: We should expect the coefficient of friction to be somewhere between 0 and 1 since this is the range of typical μ_k values. It is possible that μ_k could be greater than 1, but it can never be less than 0.

O: The easiest way to solve this problem is by considering the energy conversion for each part of the motion: gravitational potential to kinetic energy from A to B, loss of kinetic energy due to friction from B to C, and kinetic to elastic potential energy as the block compresses the spring. Choose the gravitational energy to be zero along the flat portion of the track.

A: Putting the energy equation into symbols: $U_{gA} - \left| W \right|_{BC} = U_{sf}$

expanding into specific variables: $mgy_A - f_1 d_{BC} = \frac{1}{2}\, kx_s^2$ where $f_1 = \mu_1 mg$

solving for the unknown variable: $\mu_1 mgd = mgy - \frac{1}{2}\, kx^2$ or $\mu_1 = \frac{y}{d} - \frac{kx^2}{2mgd}$

substituting: $\mu_1 = \frac{3.00\text{ m}}{6.00\text{ m}} - \frac{(2250\text{ N/m})(0.300\text{ m})^2}{2(10.0\text{ kg})(9.80\text{ m/s}^2)(6.00\text{ m})} = 0.328$

L: Our calculated value seems reasonable based on the friction data in Table 5.2. The most important aspect to solving these energy problems is considering how the energy is transferred from the initial to final energy states and remembering to subtract the energy resulting from any non-conservative forces (like friction).

8.58 The nonconservative work (due to friction) must equal the change in the kinetic energy plus the change in the potential energy.

Therefore,

$$-\mu_k mgx \cos\theta = \Delta K + \frac{1}{2}\, kx^2 - mgx \sin\theta$$

and since $v_i = v_f = 0$, $\Delta K = 0$.

Thus,

$$-\mu_k(2.00)(9.80)(\cos 37.0°)(0.200) = \frac{(100)(0.200)^2}{2} - (2.00)(9.80)(\sin 37.0°)(0.200)$$

and we find $\mu_k = \boxed{0.115}$. Note that in the above we had a *gain* in elastic potential energy for the spring and a *loss* in gravitational potential energy. The net loss in mechanical energy is equal to the energy lost due to friction.

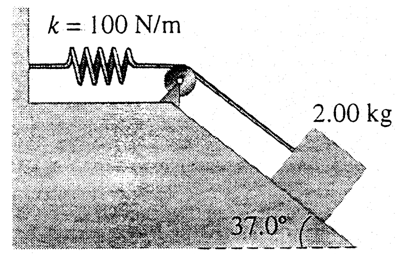
8.59 (a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg \sin \theta)x$

and $U_f = \frac{1}{2} kx^2$



Substituting values yields $(2.00)(9.80) \sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236 \text{ m}}$$

(b) $\Sigma F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg \sin \theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90 \text{ m/s}^2}$ The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.

U(spring) increases monotonically as the spring is stretched.

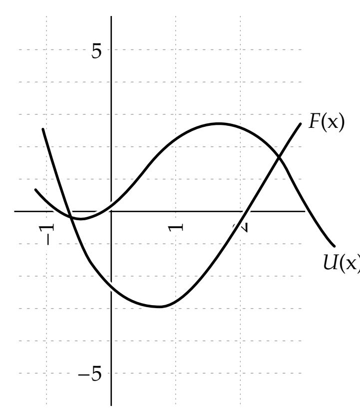
K initially increases, but then goes back to zero.

***8.60** (a) $\boxed{\mathbf{F} = -\frac{d}{dx}(-x^3 + 2x^2 + 3x)\mathbf{i} = (3x^2 - 4x - 3)\mathbf{i}}$

(b) $F = 0$ when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at $x = -0.535$ point of minimum $U(x)$

The unstable point is at $x = 1.87$ maximum in $U(x)$



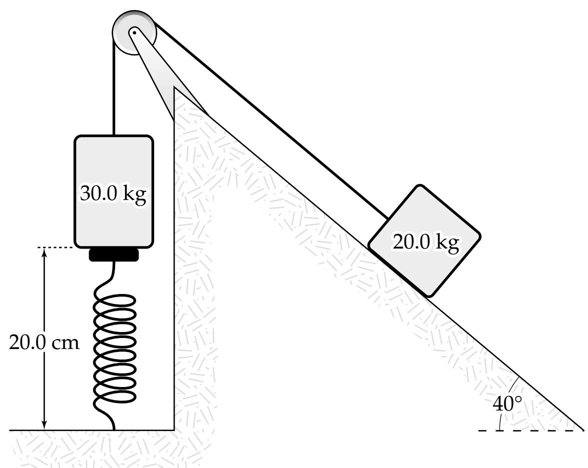
8.61 $(K + U)_i = (K + U)_f$

$$0 + (30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$= \frac{1}{2}(50.0\text{ kg})\,v^2 + (20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$



8.62 (a) Between the second and the third picture, $\Delta E = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\text{ N/m})\,d^2 + 0.250(1.00\text{ kg})(9.80\text{ m/s}^2)d - \frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s}^2) = 0$$

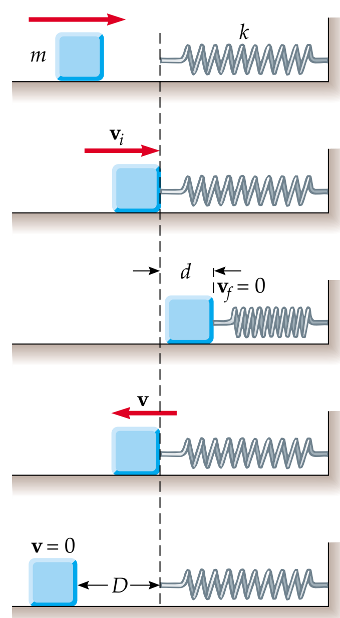
$$d = \frac{[-2.45 \pm 21.35]\text{ N}}{50.0\text{ N/m}} = \boxed{0.378\text{ m}}$$

(b) Between picture two and picture four, $\Delta E = \Delta K + \Delta U$

$$-f(2d) = -\frac{1}{2}mv^2 + \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{(1.00\text{ kg})}(2.45\text{ N})(2)(0.378\text{ m})}$$

$$= \boxed{2.30\text{ m/s}}$$

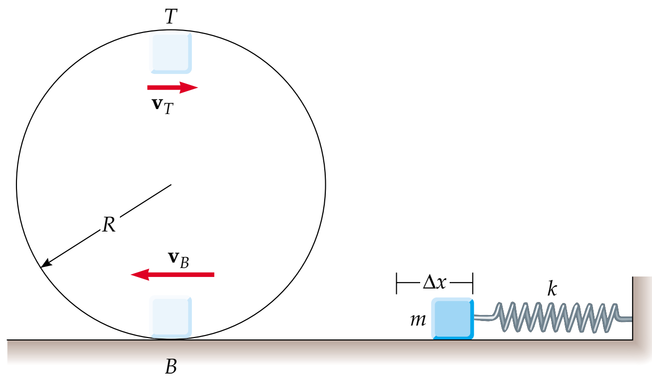


(c) For the motion from picture two to picture five, $\Delta E = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2$$

$$D = \frac{9.00\text{ J}}{2(0.250)(1.00\text{ kg})(9.80\text{ m/s}^2)} - 2(0.378\text{ m}) = \boxed{1.08\text{ m}}$$

8.63 (a)



Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2}(450\text{ N/m})(\Delta x)^2 = \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

$$\therefore\ \Delta x = \boxed{0.400\text{ m}}$$

(b) Speed of block at top of track:

$$\Delta E = W_f$$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) + \frac{1}{2}(0.500\text{ kg})\,v_T^2 - \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

$$= -(7.00\text{ N})(\pi)(1.00\text{ m})$$

$$0.250v_T^2 = 4.21 \qquad \therefore\ v_T = \boxed{4.10\text{ m/s}}$$

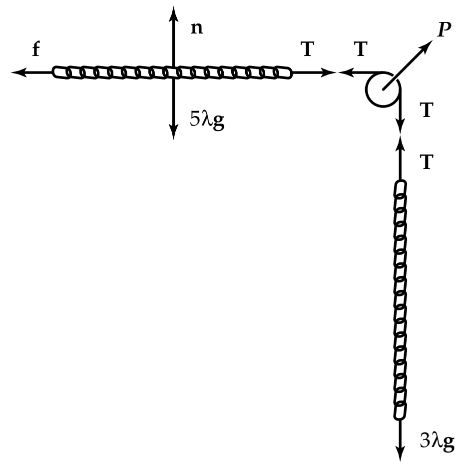
(c) Does block fall off at or before top of track?

Block falls if $a_r < g$

$$a_r = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\text{ m/s}^2$$

therefore $a_r > g$ and the $\boxed{\text{block stays on the track}}$.

8.64 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless pulley.



(a) For the five meters on the table with motion impending,

$\Sigma F_y = 0 \qquad + n - 5\lambda g = 0$

$n = 5\lambda g \qquad f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$

$\Sigma F_x = 0 \qquad + T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\Sigma F_y = 0 \qquad + T - 3\lambda g = 0 \qquad T = 3\lambda g$

so it is at this point that the chain starts to slide.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5 - x)$ remains on the table, with now

$\Sigma F_y = 0 \qquad + n - (5 - x)\lambda g = 0 \qquad n = (5 - x)\lambda g$

$f_k = \mu_k n = 0.4(5 - x)\lambda g = 2\lambda g - 0.4x\lambda g$

Consider energies at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8 - \frac{3}{2} = 6.5$ m.

$$K_i + U_i + \Delta E = K_f + U_f$$

$$0 + (m_1 g y_1 + m_2 g y_2)_i + \int_i^f f_k\, dx \cos\theta = \left(\frac{1}{2} m v^2 + m g y\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 + \int_0^5 (2\lambda g - 0.4x\lambda g)\; dx \cos 180°$$

$$= \frac{1}{2}(8\lambda)\, v^2 + (8\lambda g)4$$

$$40.0\,g + 19.5\,g - 2.00\,g\int_0^5 dx + 0.400\,g\int_0^5 x\,dx = 4.00v^2 + 32.0\,g$$

$$27.5\,g - 2.00\,gx\Big|_0^5 + 0.400\,g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5\,g - 2.00\,g(5.00) + 0.400\,g(12.5) = 4.00v^2$$

$$22.5\,g = 4.00v^2$$

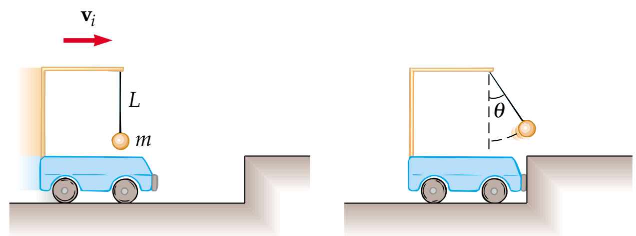
$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

8.65 (a) On the upward swing of the mass:

$$K_i + U_i + \Delta E = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + 0 + 0 = 0 + mgL(1 - \cos\theta)$$

$$v_i = \sqrt{2gL(1 - \cos\theta)}$$

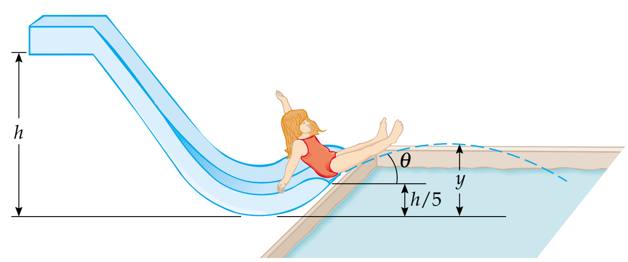


(a) (b)

(b) $v_i = \sqrt{2(9.80\text{ m/s}^2)(1.20\text{ m})(1 - \cos 35.0°)}$

$v_i = \boxed{2.06\text{ m/s}}$

8.66 Launch speed is found from



$$mg\left(\frac{4}{5}h\right) = \frac{1}{2}mv^2$$

$$v = \sqrt{2g\left(\frac{4}{5}\right)h}$$

$$v_y = v\sin\theta$$

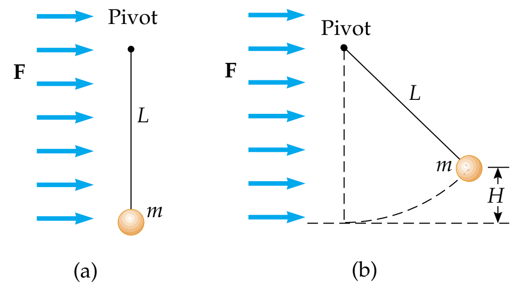
The height y above the water (by conservation of energy) is found from

$$mgy = \frac{1}{2}mv_y^2 + mg\frac{h}{5}\quad\left(\text{since } \frac{1}{2}mv_x^2 \text{ is constant in projectile motion}\right)$$

$$y = \frac{1}{2g}v_y^2 + \frac{h}{5} = \frac{1}{2g}v^2\sin^2\theta + \frac{h}{5}$$

$$y = \frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\sin^2\theta + \frac{h}{5} = \boxed{\frac{4}{5}h\sin^2\theta + \frac{h}{5}}$$

8.67 (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement



$$x = \sqrt{L^2 - (L - H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \vee\, \mathbf{F} \cdot d\mathbf{s} = F \vee dx = F\sqrt{2LH - H^2}$$

[The wind force potential energy change would be $-F\sqrt{2LH - H^2}$]

The work-energy theorem can be written:

$$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}, \text{ or}$$

$$0 + 0 + F\sqrt{2LH - H^2} = 0 + mg\,H \quad \text{giving} \quad F^2 2LH - F^2H^2 = m^2g^2H^2$$

Here $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = (F^2 + m^2g^2)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \boxed{\frac{2L}{1 + (mg/F)^2}}$$

As $F \to 0$, $H \to 0$ as is reasonable.
As $F \to \infty$, $H \to 2L$, which is unreasonable.

(b) $$H = \frac{2(2.00\text{ m})}{1 + [(2.00\text{ kg})(9.80\text{ m/s}^2)/14.7\text{ N}]^2} = \boxed{1.44\text{ m}}$$

(c) Call θ the equilibrium angle with the vertical.

$$\Sigma F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$

$$\Sigma\varpi F_y = 0 \Rightarrow T\cos\theta = mg$$

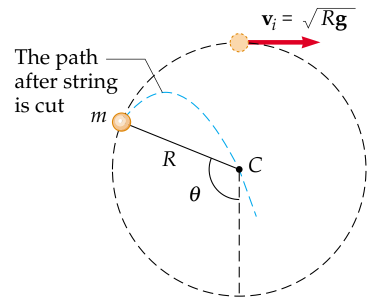
Dividing: $\tan\theta = \dfrac{F}{mg} = \dfrac{14.7\text{ N}}{19.6\text{ N}} = 0.750$, or $\theta = 36.9°$

Therefore, $H_{eq} = L(1 - \cos\theta) = (2.00\text{ m})(1 - \cos 36.9°) = \boxed{0.400\text{ m}}$

(d) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ and $H_{eq} \to\varpi\varpi L$

A very strong wind pulls the string out horizontal, parallel to the ground. Thus, $\boxed{(H_{eq})_{max} = L}$

8.68 Call $\phi = 180° - \theta$ the angle between the upward vertical and the radius to the release point. Call v_r the speed here. By conservation of energy



$$K_i + U_i + \Delta E = K_r + U_r$$

$$\frac{1}{2}mv_i^2 + mgR + 0 = \frac{1}{2}mv_r^2 + mgR\cos\phi$$

$$gR + 2gR = v_r^2 + 2gR\cos\phi$$

$$v_r = \sqrt{3gR - 2gR\cos\phi}$$

The components of velocity at release are

$$v_x = v_r\cos\phi \quad \text{and} \quad v_y = v_r\sin\phi$$

so for the projectile motion we have

$$x = v_x t \qquad R\sin\phi = v_r\cos\phi\, t$$

$$y = v_y t - \frac{1}{2}gt^2 \qquad -R\cos\phi = v_r\sin\phi\, t - \frac{1}{2}gt^2$$

By substitution

$$-R\cos\phi = v_r\sin\phi\frac{R\sin\phi}{v_r\cos\phi} - \frac{g}{2}\frac{R^2\sin^2\phi}{v_r^2\cos^2\phi}$$

with $\sin^2\phi + \cos^2\phi = 1$,

$$gR\sin^2\phi = 2v_r^2\cos\phi = 2\cos\phi(3gR - 2gR\cos\phi)$$

$$\sin^2\phi = 6\cos\phi - 4\cos^2\phi = 1 - \cos^2\phi$$

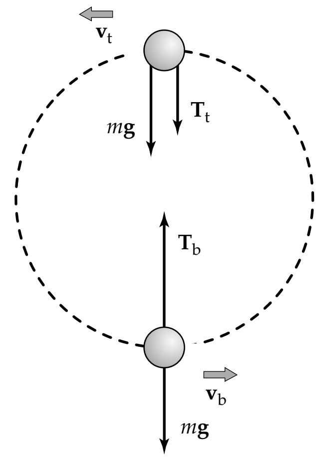
$$3\cos^2\phi - 6\cos\phi + 1 = 0$$

$$\cos\phi = \frac{6 \pm \sqrt{36-12}}{6}$$

Only the – sign gives a value for $\cos\phi$ that is less than one:

$$\cos\phi = 0.1835 \qquad \phi = 79.43° \qquad \text{so} \quad \theta = \boxed{100.6°}$$

8.69 Applying Newton's second law at the bottom (b) and top (t) of the circle gives



$$T_b - mg = \frac{mv_b^2}{R} \quad \text{and} \quad -T_t - mg = -\frac{mv_t^2}{R}$$

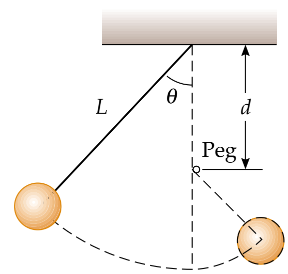
Adding these gives $\quad T_b = T_t + 2mg + \dfrac{m(v_b^2 - v_t^2)}{R}$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\dfrac{m(v_b^2 - v_t^2)}{2} + (0 - 2mgR) = 0 \quad$ and $\quad \dfrac{m(v_b^2 - v_t^2)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$

8.70 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.



(b) Relative to the point of suspension,

$$U_i = 0, \quad U_f = -mg[d - (L - d)]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}\,mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \quad \text{where } R = L - d.$$

Upon solving, we get $\boxed{d = \dfrac{3L}{5}}$

8.71 (a) The potential energy associated with the wind force is $+Fx$, where x is the horizontal distance traveled, with x positive when swinging into the wind and negative when swinging in the direction the wind is blowing. The initial energy of Jane is, (using the pivot point of the swing as the point of zero gravitational energy),

$$E_i = (K + U_g + U_{\text{wind}})_i = \frac{1}{2}\, mv_i^2 - mgL\cos\theta - FL\sin\theta$$

where m is her mass. At the end of her swing, her energy is

$$E_f = (K + U_g + U_{\text{wind}})_f = 0 - mgL\cos\phi + FL\sin\phi$$

so conservation of energy ($E_i = E_f$) gives

$$\frac{1}{2}\, mv_i^2 - mgL\cos\theta - FL\sin\theta = -mgL\cos\phi + FL\sin\phi$$

This leads to $v_i = \sqrt{2gL(\cos\theta - \cos\phi) + 2\dfrac{FL}{m}(\sin\theta + \sin\phi)}$

But $D = L\sin\phi + L\sin\theta$, so that $\sin\phi = \dfrac{D}{L} - \sin\theta = \dfrac{50.0}{40.0} - \sin 50.0° = 0.484$

which gives $\phi = 28.9°$. Using this, we have $\boxed{v_i = 6.15\ \text{m/s}}$.

(b) Here (again using conservation of energy) we have,

$$-MgL\cos\phi + FL\sin\phi + \frac{1}{2}\, Mv^2 = -MgL\cos\theta - FL\sin\theta$$

where M is the combined mass of Jane and Tarzan.

Therefore, $v = \sqrt{2gL(\cos\phi - \cos\theta) - 2\dfrac{FL}{M}(\sin\phi + \sin\theta)}$ which gives $\boxed{v = 9.87\ \text{m/s}}$ as the minimum speed needed.

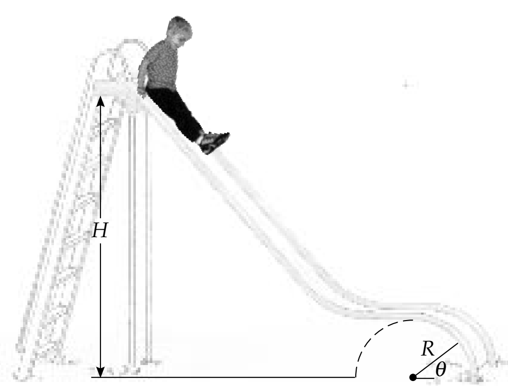
8.72 Find the velocity at the point where the child leaves the slide, height h:

$$(U + K)_i = (U + K)_f$$

$$mgH + 0 = mgh + \frac{1}{2}\, mv^2$$

$$v = \sqrt{2g(H - h)}$$

Use Newton's laws to compare h and H.



(Recall the normal force will be zero):

$$\sum F_r = ma_r = \frac{mv^2}{R}$$

$$mg \sin \theta - n = \frac{mv^2}{R}$$

$$mg \sin \theta = \frac{m(2\,g)(H-h)}{R}$$

Put θ in terms of R: $\sin \theta = \frac{h}{R}$

$$mg\left(\frac{h}{R}\right) = \frac{2\,mg(H-h)}{R}$$

$$\boxed{h = \frac{2}{3}H}$$

Notice if $H \geq \frac{3}{2}\;R$, the assumption that the child will leave the slide at a height $\frac{2}{3}\;H$ is no longer valid. Then the velocity will be too large for the centripetal force to keep the child on the slide. Thus if $H \geq \frac{3}{2}\;R$, the child will leave the track at $h = R$.

8.73 Case I: Surface is frictionless

$$\frac{1}{2}\;mv^2 = \frac{1}{2}\;kx^2$$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20 \infty 10^2\text{ N/m}$$

Case II: Surface is rough, $\mu_k = 0.300$

$$\frac{1}{2}\;mv^2 = \frac{1}{2}\;kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}\;v^2 = \frac{1}{2}(7.20 \times 10^2\text{ N}/m)(10^{-1}\;m)^2 - (0.300)(5.00\text{ kg})(9.80\text{ m/s}^2)(10^{-1}\text{ m})$$

$$\boxed{v = 0.923\text{ m/s}}$$

8.74 $\Sigma F_y = n - mg \cos 37.0° = 0, \quad \therefore\; n = mg \cos 37.0° = 400\text{ N}$

$f = \mu N = (0.250)(400) = 100\text{ N}$

$W_f = \Delta E$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0 \sin 37.0°) = 5.90 \infty 10^3$

$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96 \infty 10^4$

$\Delta K_A = \frac{1}{2} m_A(v_f^2 - v_i^2)$

$\Delta K_B \ \frac{1}{2} m_B(v_f^2 - v_i^2) = \frac{m_B}{m_A} \Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A = \boxed{3.92 \text{ kJ}}$