Chapter 4 Solutions

***4.1**

x(m)	y(m)
0	– 3600
– 3000	0
– 1270	1270
– 4270 m	– 2330 m

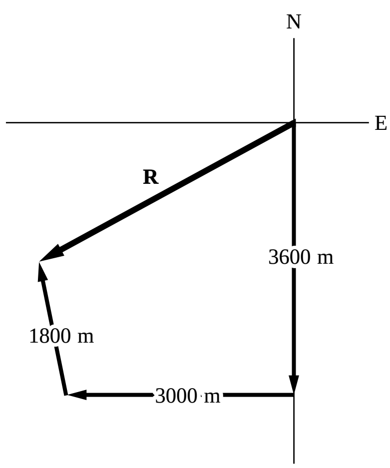
(a) Net displacement $= \sqrt{x^2 + y^2}$

$$= \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) Average speed $= \dfrac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{(180 \text{ s} + 120 \text{ s} + 60.0 \text{ s})}$

$$= \boxed{23.3 \text{ m/s}}$$

(c) Average velocity $= \dfrac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \mathbf{R}}$



4.2 (a) For the average velocity, we have

$$\overline{\mathbf{v}} = \left(\frac{x(4.00) - x(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\mathbf{i} + \left(\frac{y(4.00) - y(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\mathbf{j}$$

$$= \left(\frac{5.00 \text{ m} - 3.00 \text{ m}}{2.00 \text{ s}}\right)\mathbf{i} + \left(\frac{3.00 \text{ m} - 1.50 \text{ m}}{2.00 \text{ s}}\right)\mathbf{j}$$

$$\boxed{\overline{\mathbf{v}} = (1.00\mathbf{i} + 0.750\mathbf{j}) \text{ m/s}}$$

(b) For the velocity components, we have

$$v_x = \frac{dx}{dt} = a = 1.00 \text{ m/s}$$

$$v_y = \frac{dy}{dt} = 2ct = (0.250 \text{ m/s}^2)t$$

Therefore, $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = (1.00 \text{ m/s})\mathbf{i} + (0.250 \text{ m/s}^2)\, t\mathbf{j}$

$$\boxed{\mathbf{v}(2.00) = (1.00 \text{ m/s})\mathbf{i} + (0.500 \text{ m/s})\mathbf{j}}$$

and the speed is

$$\left|\mathbf{v}\right|_{t = 2.00 \text{ s}} = \sqrt{(1.00 \text{ m/s})^2 + (0.500 \text{ m/s})^2} = \boxed{1.12 \text{ m/s}}$$

4.3 (a) $\mathbf{r} = \boxed{18.0t\ \mathbf{i} + (4.00t - 4.90t^2)\mathbf{j}}$

(b) $\mathbf{v} = \boxed{(18.0 \text{ m/s})\mathbf{i} + [4.00 \text{ m/s} - (9.80 \text{ m/s}^2)t]\mathbf{j}}$

(c) $\mathbf{a} = \boxed{(-9.80 \text{ m/s}^2)\mathbf{j}}$

(d) $\mathbf{r}(3.00 \text{ s}) = \boxed{(54.0 \text{ m})\mathbf{i} - (32.1 \text{ m})\mathbf{j}}$

(e) $\mathbf{v}(3.00 \text{ s}) = \boxed{(18.0 \text{ m/s})\mathbf{i} - (25.4 \text{ m/s})\mathbf{j}}$

(f) $\mathbf{a}(3.00 \text{ s}) = \boxed{(-9.80 \text{ m/s}^2)\mathbf{j}}$

4.4 (a) From $x = -5.00 \sin\omega t$, the x-component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\omega \sin\omega t) = -5.00\omega \cos\omega t$$

and $\quad a_x = \dfrac{dv_x}{dt} = +5.00\omega^2 \sin\omega t$

similarly, $v_y = \left(\dfrac{d}{dt}\right)(4.00 - 5.00 \cos\omega t) = 0 + 5.00\omega \sin\omega t$

and $a_y = \left(\dfrac{d}{dt}\right)(5.00\omega \sin\omega t) = 5.00\omega^2 \cos\omega t$

At $t = 0$, $\mathbf{v} = -5.00\omega \cos 0\ \mathbf{i} + 5.00\omega \sin 0\ \mathbf{j} = \boxed{(-5.00\omega\ \mathbf{i} + 0\ \mathbf{j}) \text{ m/s}}$

and $\mathbf{a} = 5.00\omega^2 \sin 0\ \mathbf{i} + 5.00\omega^2 \cos 0\ \mathbf{j} = \boxed{(0\ \mathbf{i} + 5.00\omega^2\ \mathbf{j}) \text{ m/s}^2}$

(b) $\mathbf{r} = x\mathbf{i} + y\mathbf{j} = \boxed{(4.00\text{ m})\mathbf{j} + (5.00\text{ m})(-\sin\omega t\,\mathbf{i} - \cos\omega t\,\mathbf{j})}$

$\mathbf{v} = \boxed{(5.00\text{ m})\omega[-\cos\omega t\,\mathbf{i} + \sin\omega t\,\mathbf{j}]}$

$a = \boxed{(5.00\text{ m})\omega^2[\sin\omega t\,\mathbf{i} + \cos\omega t\,\mathbf{j}]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at (0, 4.00 m)}}$.

4.5 (a) $\mathbf{v} = \mathbf{v}_i + \mathbf{a}t$

$$\mathbf{a} = \frac{(\mathbf{v} - \mathbf{v}_i)}{t} = \frac{[(9.00\mathbf{i} + 7.00\mathbf{j}) - (3.00\mathbf{i} - 2.00\mathbf{j})]}{3.00} = \boxed{(2.00\mathbf{i} + 3.00\mathbf{j})\text{ m/s}^2}$$

(b) $\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t \frac{1}{2}\mathbf{a}t^2 = (3.00\mathbf{i} - 2.00\mathbf{j})t + \frac{1}{2}(2.00\mathbf{i} + 3.00\mathbf{j})\,t^2$;

$\boxed{x = (3.00t + t^2)\text{ m}}$ and $\boxed{y = (1.50t^2 - 2.00t)\text{ m}}$

4.6 (a) $$v = \frac{dr}{dt} = \left(\frac{d}{dt}\right)(3.00\mathbf{i} - 6.00t^2\mathbf{j}) = \boxed{-12.0t\mathbf{j}\text{ m/s}}$$

$$a = \frac{dv}{dt} = \left(\frac{d}{dt}\right)(-12.0t\mathbf{j}) = \boxed{-12.0\mathbf{j}\text{ m/s}^2}$$

(b) $\boxed{\mathbf{r} = (3.00\mathbf{i} - 6.00\mathbf{j})\text{ m};\ \mathbf{v} = -12.0\mathbf{j}\text{ m/s}}$

4.7 $\mathbf{v}_i = (4.00\mathbf{i} + 1.00\mathbf{j})$ m/s and $\mathbf{v}(20.0) = (20.0\mathbf{i} - 5.00\mathbf{j})$ m/s.

(a) $$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0 - 4.00}{20.0}\text{ m/s}^2 = \boxed{0.800\text{ m/s}^2}$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00 - 1.00}{20.0}\text{ m/s}^2 = \boxed{-0.300\text{ m/s}^2}$$

(b) $$\theta = \tan^{-1}\left[\frac{-0.300}{0.800}\right] = -20.6° = \boxed{339°\text{ from }+x\text{ axis}}$$

(c) At $t = 25.0$ s,

$$x = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360\text{ m}}$$

$$y = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

***4.8** $\mathbf{a} = 3.00\mathbf{j}\ \mathrm{m/s^2}$; $\mathbf{v}_\mathrm{i} = 5.00\mathbf{i}\ \mathrm{m/s}$; $\mathbf{r}_\mathrm{i} = 0\mathbf{i} + 0\mathbf{j}$

(a) $$\mathbf{r} = \mathbf{r}_\mathrm{i} + \mathbf{v}_\mathrm{i}t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\mathbf{i} + \frac{1}{2}3.00t^2\mathbf{j}\right]\mathrm{m}}$$

$$\mathbf{v} = \mathbf{v}_\mathrm{i} + \mathbf{a}t = \boxed{(5.00\mathbf{i} + 3.00t\mathbf{j})\ \mathrm{m/s}}$$

(b) $$t = 2.00\ \mathrm{s},\ \mathbf{r} = (5.00)(2.00)\mathbf{i} + \frac{1}{2}(3.00)(2.00)^2\mathbf{j} = (10.0\mathbf{i} + 6.00\mathbf{j})\ \mathrm{m}$$

so $x = \boxed{10.0\ \mathrm{m}}$, $y = \boxed{6.00\ \mathrm{m}}$

$$\mathbf{v} = 5.00\mathbf{i} + (3.00)(2.00)\mathbf{j} = (5.00\mathbf{i} + 6.00\mathbf{j})\ \mathrm{m/s}$$

$$v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \mathrm{m/s}}$$

4.9 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x = v_{xi}t$. i.e., $t = \dfrac{x}{v_{xi}} = \dfrac{(1.40\ \mathrm{m})}{v_{xi}}$. In the same time it falls a distance 0.860 m with acceleration downward of $9.80\ \mathrm{m/s^2}$. Then using

$$y = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

we have

$$0 = 0.860\ \mathrm{m} - \frac{1}{2}(9.80\ \mathrm{m/s^2})\left(\frac{1.40\ \mathrm{m}}{v_{xi}}\right)^2$$

i.e., $$v_{xi} = \sqrt{\frac{(4.90\ \mathrm{m/s^2})(1.96\ \mathrm{m^2})}{0.860\ \mathrm{m}}} = \boxed{3.34\ \mathrm{m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_y = v_{yi} + a_y t = -(9.80\ \mathrm{m/s^2})\left(\frac{1.40\ \mathrm{m}}{3.34\ \mathrm{m/s}}\right) = -4.11\ \mathrm{m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

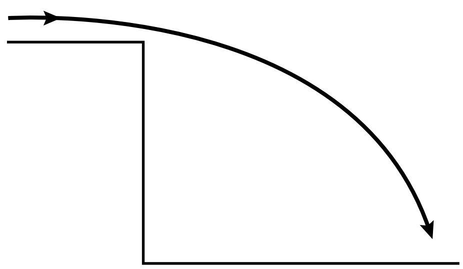
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}$$

Goal Solution

G: Based on our everyday experiences and the description of the problem, a reasonable speed of the mug would be a few m/s and it will hit the floor at some angle between 0 and 90°, probably about 45°.

O: We are looking for two different velocities, but we are only given two distances. Our approach will be to separate the vertical and horizontal motions. By using the height that the mug falls, we can find the time of the fall. Once we know the time, we can find the horizontal and vertical components of the velocity. For convenience, we will set the origin to be the point where the mug leaves the counter.

A: Vertical motion: $y = -0.860$ m, $v_{yi} = 0$, v_y = unknown, $a_y = -9.80$ m/s^2
Horizontal motion: $x = 1.40$ m, v_x = constant (unknown), $a_x = 0$



(a) To find the time of fall, we use the free fall equation: $y = v_{yi}t + \frac{1}{2}a_y t^2$

Solving: $-0.860 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$ so that $t = 0.419$ s

Then $v_x = \dfrac{x}{t} = \dfrac{1.40 \text{ m}}{0.419 \text{ s}} = 3.34$ m/s

(b) As the mug hits the floor, $v_y = v_{yi} + a_y t = 0 - (9.8 \text{ m/s}^2)(0.419 \text{ s}) = -4.11$ m/s

The impact angle is $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left(\dfrac{4.11 \text{ m/s}}{3.34 \text{ m/}}\right) = 50.9°$ below the horizontal

L: This was a multi-step problem that required several physics equations to solve; our answers do agree with our initial expectations. Since the problem did not ask for the time, we could have eliminated this variable by substitution, but then we would have had to substitute the algebraic expression $t = 2y/g$ into two other equations. So in this case it was easier to find a numerical value for the time as an intermediate step. Sometimes the most efficient method is not realized until each alternative solution is attempted.

***4.10** The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \text{ and } y = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

When the mug reaches the floor, $y = -h$, so $-h = -\frac{1}{2}gt^2$ which gives the time of impact as $t = \sqrt{\frac{2h}{g}}$.

(a) Since $x = d$ when the mug reaches the floor, $x = v_{xi}t$ becomes

$$d = v_{xi}\sqrt{\frac{2h}{g}}$$

giving the initial velocity as $\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$.

(b) Just before impact, the x-component of velocity is still $v_{xf} = v_{xi}$ while the y-component is $v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}$. Then the direction of motion just before impact is below the horizontal at an angle of $\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right)$, or

$$\theta = \tan^{-1}\left(g\sqrt{\frac{2h}{g}} \Big/ d\sqrt{\frac{g}{2h}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}$$

4.11 (a) The time of flight of the first snowball is the nonzero root of

$$y = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$$

$$0 = 0 + (25.0 \text{ m/s}) \sin 70.0° t_1 - \frac{1}{2}(9.80 \text{ m/s}^2)\, t_1^2$$

$$t_1 = \frac{2(25.0 \text{ m/s}) \sin 70.0°}{9.80 \text{ m/s}^2} = 4.79 \text{ s}$$

The distance to your target is

$$x - x_i = v_{xi}t_1 = (25.0 \text{ m/s}) \cos 70.0° \,(4.79 \text{ s}) = 41.0 \text{ m}$$

Now the second snowball we describe by

$$y = y_i + v_{yi}t_2 + \frac{1}{2}\ a_y t_2^2$$

$$0 = (25.0 \text{ m/s}) \sin \theta_2 t_2 - (4.90 \text{ m/s}^2) t_2^2$$

$$t_2 = (5.10 \text{ s}) \sin \theta_2$$

$$x - x_i = v_{xi}t_2$$

$$41.0 \text{ m} = (25.0 \text{ m/s}) \cos \theta_2 (5.10 \text{ s}) \sin \theta_2 = (128 \text{ m}) \sin \theta_2 \cos \theta_2$$

$$0.321 = \sin \theta_2 \cos \theta_2$$

Using $\sin 2\theta = 2 \sin \theta \cos \theta$ we can solve

$$0.321 = \frac{1}{2} \sin 2\theta_2 \qquad 2\theta_2 = \text{Arcsin } 0.643 \qquad \boxed{\theta_2 = 20.0°}$$

(b) The second snowball is in the air for time $t_2 = (5.10 \text{ s}) \sin \theta_2 = (5.10 \text{ s}) \sin 20.0° = 1.75 \text{ s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79 \text{ s} - 1.75 \text{ s} = \boxed{3.05 \text{ s}}\ .$$

***4.12** $y = v_i (\sin 3.00°)t - \frac{1}{2}\ gt^2$, $v_y = v_i \sin 3.00° - gt$

When $y = 0.330$ m, $v_y = 0$ and $v_i \sin 3.00° = gt$

$$y = v_i (\sin 3.00°) \frac{v_i \sin 3.00°}{g} - \frac{1}{2}\ g\left(\frac{v_i \sin 3.00°}{g}\right)^2$$

$$y = \frac{v_i^2 \sin^2 3.00°}{2g} = 0.330 \text{ m}$$

$$\therefore v_i = \boxed{48.6 \text{ m/s}}$$

The 12.6 m is unnecessary information.

***4.13** $x = v_{xi}t = v_i \cos \theta_i t$

$$x = (300 \text{ m/s})(\cos 55.0°)(42.0 \text{ s})$$

$$x = \boxed{7.23 \times 10^3 \text{ m}}$$

$$y = v_{yi}t - \frac{1}{2}\ gt^2 = v_i\ \sin \theta_i t - \frac{1}{2}\ gt^2$$

$$y = (300 \text{ m/s})(\sin 55.0°)(42.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(42.0 \text{ s})^2 = \boxed{1.68 \times 10^3 \text{ m}}$$

***4.14** From Equation 4.14,

$$R = 15.0\text{ m},\ \ v_i = 3.00\text{ m/s},\ \ \theta_{\max} = 45.0°$$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\text{ m/s}^2}$$

4.15 $h = \dfrac{v_i^2 \sin^2\theta_i}{2g}$; $R = \dfrac{v_i^2(\sin 2\theta_i)}{g}$; $3h = R$,

so $$\frac{3v_i^2\sin^2\theta_i}{2g} = \frac{v_i^2(\sin 2\theta_i)}{g}$$

or $$\frac{2}{3} = \frac{\sin^2\theta_i}{\sin 2\theta_i} = \frac{\tan\theta_i}{2}$$

thus $\theta_i = \tan^{-1}\left(\dfrac{4}{3}\right) = \boxed{53.1°}$

4.16 (a) $x = v_{xi}t = (8.00\cos 20.0°)(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards,

$$y = v_{yi}t + \frac{1}{2}gt^2$$

$$= 8.00(\cos 20.0°)3.00 + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}$$

(c) $10.0 = 8.00\cos 20.0°\ t + \dfrac{1}{2}(9.80)\ t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

4.17 $x = v_{xi}t$

$2000 \text{ m} = (1000 \text{ m/s}) \cos \theta_i t$

$$t = \frac{2.00 \text{ s}}{\cos \theta_i}$$

$$y = v_{yi}t + \frac{1}{2} a_y t^2$$

$$800 \text{ m} = (1000 \text{ m/s}) \sin \theta_i t - \frac{1}{2}(9.80 \text{ m/s}^2)\, t^2$$

$$800 \text{ m} = (1000 \text{ m/s}) \sin \theta_i \left(\frac{2.00 \text{ s}}{\cos \theta_i}\right) - \frac{1}{2}(9.80 \text{ m/s}^2)\left(\frac{2.00 \text{ s}}{\cos \theta_i}\right)^2$$

$$800 \text{ m} \cos^2 \theta_i = (2000 \text{ m}) \sin \theta_i \cos \theta_i - 19.6 \text{ m}$$

$$19.6 \text{ m} + 800 \text{ m} \cos^2 \theta_i = 2000 \text{ m} \sqrt{1 - \cos^2 \theta_i}\, \cos \theta_i$$

$$384 \text{ m}^2 + 31\,360 \text{ m}^2 \cos^2 \theta_i + 640\,000 \text{ m}^2 \cos^4 \theta_i$$

$$= 4\,000\,000 \text{ m}^2 \cos^2 \theta_i - 4\,000\,000 \text{ m}^2 \cos^4 \theta_i$$

$$4\,640\,000 \cos^4 \theta_i - 3\,968\,640 \cos^2 \theta_i + 384 = 0$$

$$\cos^2 \theta_i = \frac{3\,968\,640 \pm \sqrt{(3\,968\,640)^2 - 4(4\,640\,000)(384)}}{9\,280}$$

$\cos \theta_i = 0.925$ or 0.00984

$\theta_i = \boxed{22.4° \quad \text{or} \quad 89.4°}$ Both solutions are valid.

***4.18** The equation $y = (\tan \theta_i)x - \left(\dfrac{g}{2v_i^2 \cos^2 \theta_i}\right) x^2$ describes the trajectory of the projectile. When y is a maximum (at $x = x_h$), the slope is zero $\left(\text{ie., } \dfrac{dy}{dx} = 0 \text{ at } x = x_h\right)$. This gives $\left(\dfrac{dy}{dx}\right)_{x = x_h} = \tan \theta_i - \left(\dfrac{g}{2v_i^2 \cos^2 \theta_i}\right) 2x_h = 0$, so the x-coordinate at which the maximum height occurs is $\boxed{x_h = \dfrac{v_i^2 \sin \theta_i \cos \theta_i}{g}}$. The maximum-height point is halfway through the entire symmetrical trajectory. Thus, the horizontal range is $R = 2x_h = \dfrac{v_i^2\, 2 \sin \theta_i \cos \theta_i}{g} = \boxed{\dfrac{v_i^2 \sin 2\theta_i}{g}}$.

4.19 (a) We use Equation 4.12:

$$y = x \tan \theta_i - \frac{gx^2}{2v_i^2 \cos^2 \theta_i}$$

With x = 36.0 m, v_i = 20.0 m / s, and θ = 53.0°, we find

$$y = (36.0 \text{ m})(\tan 53.0°) - \frac{(9.80 \text{ m/s}^2)(36.0 \text{ m})^2}{(2)(20.0 \text{ m/s})^2 \cos^2 53.0°} = 3.94 \text{ m}$$

The ball clears the bar by (3.94 – 3.05) m = $\boxed{0.889 \text{ m}}$.

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin \theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

The time to travel 36.0 m horizontally is $t_2 = \dfrac{x}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

4.20 (40.0 m / s)(cos 30.0°)t = 50.0 m. (Eq. 4.10)

The stream of water takes t = 1.44 s to reach the building, which it strikes at a height

$$y = v_{yi}t - \frac{1}{2} gt^2$$

$$= (40.0 \sin 30.0°)t - \frac{1}{2}(9.80)\, t^2 = (40.0)\left(\frac{1}{2}\right)(1.44) - (4.90)(1.44)^2 = \boxed{18.7 \text{ m}}$$

4.21 From Equation 4.10, $x = v_{xi}t = (v_i \cos \theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \dfrac{d}{v_i \cos \theta_i}$. At this time, the altitude of the water is

$$y = v_{yi}t + \frac{1}{2} a_y t^2 = (v_i \sin \theta_i)\left(\frac{d}{v_i \cos \theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos \theta_i}\right)^2$$

Therefore the water strikes the building at a height of $\boxed{y = d \tan \theta_i - \dfrac{gd^2}{2v_i^2 \cos^2 \theta_i}}$ above ground level.

4.22 The horizontal kick gives zero vertical velocity to the ball. Then its time of flight follows from

$$y = y_i + v_{yi}t + \frac{1}{2}\ a_y t^2$$

$$-40.0\text{ m} = 0 + 0 + \frac{1}{2}(-9.80\text{ m/s}^2)\ t^2$$

$$t = 2.86\text{ s}$$

The extra time 3.00 s – 2.86 s = 0.143 s is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343\text{ m/s})0.143\text{ s} = 49.0\text{ m} = \sqrt{x^2 + (40.0\text{ m})^2}$$

where x represents the horizontal distance the ball travels.

$$x = 28.3\text{ m} = v_{xi}t + 0t^2$$

$$\therefore\ v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

***4.23** From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\text{ m/s}^2)(1.85 - 1.02)\text{ m}$. From this, $v_{yi} = 4.03\text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation becomes

$$v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.900 - 1.85)\text{ m, giving } v_{yf} = -4.32\text{ m/s}$$

as the vertical velocity just before he lands.

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$ as follows:

$$-4.32\text{ m/s} = 4.03\text{ m/s} + (-9.80\text{ m/s}^2)t$$

or $\boxed{t = 0.852\text{ s}}$

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes $2.80\text{ m} = v_{xi}(0.852\text{ s})$, which yields $\boxed{v_{xi} = 3.29\text{ m/s}}$.

(c) $\boxed{v_{yi} = 4.03\text{ m/s}}$ See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\frac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\frac{4.03\text{ m/s}}{3.29\text{ m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i):\ 0 = v_{yi}^2 + 2(-9.80\text{ m/s}^2)(2.50 - 1.20)\text{ m}$$

so $v_{yi} = 5.04\text{ m/s}$

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields

$$v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.700 - 2.50)\text{ m}$$

and $v_{yf} = -5.94\text{ m/s}$

The hang time is then found as

$$v_{yf} = v_{yi} + a_y t:\ -5.94\text{ m/s} = 5.04\text{ m/s} + (-9.80\text{ m/s}^2)t$$

and $\boxed{t = 1.12\text{ s}}$

4.24 (a) $v = \frac{\Delta x}{\Delta t} = \frac{2\pi(3.84 \times 10^8\text{ m})}{[(27.3\text{ d})(24\text{ h/d})(3600\text{ s/h})]} = \boxed{1.02 \times 10^3\text{ m/s}}$

(b) Since v is constant and only direction changes,

$$a = \frac{v^2}{r} = \frac{(1.02 \times 10^3)^2}{(3.84 \times 10^8)} = \boxed{2.72 \times 10^{-3}\text{ m/s}^2}$$

4.25 $a_r = \frac{v^2}{r} = \frac{(20.0\text{ m/s})^2}{(1.06\text{ m})} = \boxed{377\text{ m/s}^2}$

The mass is unnecessary information.

4.26 $a = \frac{v^2}{R}$ $\qquad T = (24\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right) = 86400\text{ s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37 \times 10^6\text{ m})}{86400\text{ s}} = 463\text{ m/s}$$

$$a = \frac{(463\text{ m/s})^2}{6.37 \times 10^6\text{ m}} = \boxed{0.0337\text{ m/s}^2}$$ (directed toward the center of the Earth)

4.27 $r = 0.500\text{ m};\ v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\text{ m})}{(60.0\text{ s}/200\text{ rev})} = 10.47\text{ m/s}$ $\qquad \boxed{10.5\text{ m/s}}$

$$a = \frac{v^2}{r} = \frac{(10.47)^2}{0.5} = \boxed{219\text{ m/s}^2\text{ (inward)}}$$

***4.28** The centripetal acceleration is $a_r = \dfrac{v^2}{r}$, so the required speed is

$$v = \sqrt{a_r r} = \sqrt{1.40(9.80 \text{ m/s}^2)(10.0 \text{ m})} = 11.7 \text{ m/s}$$

The period (time for one rotation) is given by $T = 2\pi r / v$ and the rotation rate is the frequency:

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{11.7 \text{ m/s}}{2\pi(10.0 \text{ m})} = \boxed{0.186 \text{ s}^{-1}}$$

4.29. (a) $v = r\omega$

At 8.00 rev/s, $v = (0.600 \text{ m})(8.00 \text{ rev/s})(2\pi \text{ rad/rev}) = 30.2 \text{ m/s} = 9.60\pi \text{ m/s}$

At 6.00 rev/s, $v = (0.900 \text{ m})(6.00 \text{ rev/s})(2\pi \text{ rad/rev}) = 33.9 \text{ m/s} = 10.8\pi \text{ m/s}$

$\boxed{6.00 \text{ rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi \text{ m/s})^2}{0.600 \text{ m}} = \boxed{1.52 \times 10^3 \text{ m/s}^2}$

(c) At 6.00 rev/s, acceleration $= \dfrac{(10.8\pi \text{ m/s})^2}{0.900 \text{ m}} = \boxed{1.28 \times 10^3 \text{ m/s}^2}$

***4.30.** The satellite is in free fall. Its acceleration is due to the acceleration of gravity and is by effect a centripetal acceleration:

$$a_r = g$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{rg} = \sqrt{(6400 + 600)(10^3 \text{ m})(8.21 \text{ m/s}^2)} = \boxed{7.58 \times 10^3 \text{ m/s}}$$

$$v = \frac{2\pi r}{T} \quad \text{and} \quad T = \frac{2\pi r}{v} = \frac{2\pi(7000 \times 10^3 \text{ m})}{(7.58 \times 10^3 \text{ m/s})} = \boxed{5.80 \times 10^3 \text{ s}}$$

$$T = (5.80 \times 10^3 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 96.7 \text{ min}$$

4.31 We assume the train is still slowing down at the instant in question.

$$a_r = \frac{v^2}{r} = 1.29\ \text{m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0\ \text{km/h})(10^3\ \text{m/km})(1\ \text{h}/3600\ \text{s})}{15.0\ \text{s}} = -0.741\ \text{m/s}^2$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.29\ \text{m/s}^2)^2 + (-0.741\ \text{m/s}^2)^2}$$

$$= \boxed{1.48\ \text{m/s}^2 \text{ inward and } 29.9° \text{ backward}}$$

Goal Solution

G: If the train is taking this turn at a safe speed, then its acceleration should be significantly less than g, perhaps a few m/s^2 (otherwise it might jump the tracks!), and it should be directed toward the center of the curve and backward since the train is slowing.

O: Since the train is changing both its speed and direction, the acceleration vector will be the vector sum of the tangential and radial acceleration components. The tangential acceleration can be found from the changing speed and elapsed time, while the radial acceleration can be found from the radius of curvature and the train's speed.

A: First, let's convert the speeds to units from km/h to m/s:

$$v_i = 90.0\ \text{km/h} = (90.0\ \text{km/h})\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 25.0\ \text{m/s}$$

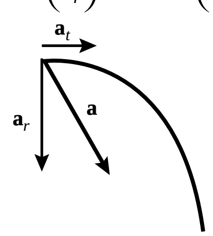
$$v_f = 50.0\ \text{km/h} = (50.0\ \text{km/h})\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 13.9\ \text{m/s}$$

Tangential accel.: $a_t = \dfrac{\Delta v}{\Delta t} = \dfrac{13.9\ \text{m/s} - 25.0\ \text{m/s}}{15.0\ \text{s}} = -0.741\ \text{m/s}^2$ (backward)

Radial acceleration: $a_r = \dfrac{v^2}{r} = \dfrac{(13.9\ \text{m/s})^2}{150\ \text{m}} = 1.29\ \text{m/s}^2$ (inward)

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(-0.741\ \text{m/s}^2)^2 + (1.29\ \text{m/s}^2)^2} = 1.48\ \text{m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{0.741\ \text{m/s}^2}{1.29\ \text{m/s}^2}\right) = 29.9° \text{ (backwards from a radial line)}$$



L: The acceleration is clearly less than g, and it appears that most of the acceleration comes from the radial component, so it makes sense that the acceleration vector should point mostly toward the center of the curve and slightly backwards due to the negative tangential acceleration.

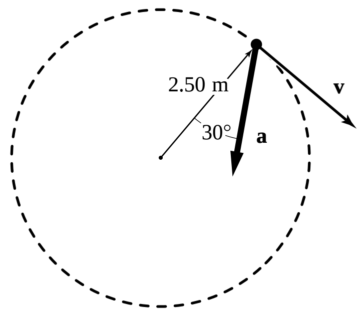
***4.32** (a) $a_t = \boxed{0.600 \text{ m/s}^2}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(4.00 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{0.800 \text{ m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00 \text{ m/s}^2}$

$\theta = \tan^{-1}\dfrac{a_r}{a_t} = \boxed{53.1° \text{ inward from path}}$

4.33 $r = 2.50$ m, $a = 15.0 \text{ m/s}^2$



(a) $a_r = a \cos 30.0° = (15.0 \text{ m/s}^2) \cos 30.0° = \boxed{13.0 \text{ m/s}^2}$

(b) $a_r = \dfrac{v^2}{r}$

so $v^2 = ra_r = (2.50 \text{ m})(13.0 \text{ m/s}^2) = 32.5 \text{ m}^2/\text{s}^2$

$v = \sqrt{32.5} \text{ m/s} = \boxed{5.70 \text{ m/s}}$

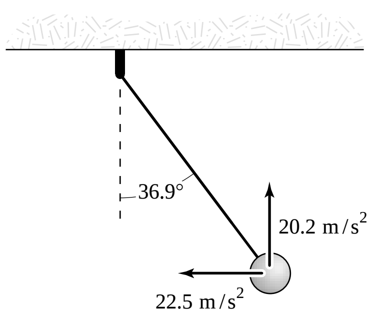
(c) $a^2 = a_t^2 + a_r^2$ so

$a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0 \text{ m/s}^2)^2 - (13.0 \text{ m/s}^2)^2} = \boxed{7.50 \text{ m/s}^2}$

4.34 (a) $a_{\text{top}} = \dfrac{v^2}{r} = \dfrac{(4.30 \text{ m/s})^2}{0.600 \text{ m}} = \boxed{30.8 \text{ m/s}^2 \text{ down}}$

(b) $a_{\text{bottom}} = \dfrac{v^2}{r} = \dfrac{(6.50 \text{ m/s})^2}{0.600 \text{ m}} = \boxed{70.4 \text{ m/s}^2 \text{ upward}}$

4.35 (a)



(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the radial acceleration:

$$a_r = (22.5 \text{ m/s}^2) \cos (90.0° - 36.9°) + (20.2 \text{ m/s}^2) \cos 36.9°$$

$a_r = \boxed{29.7 \text{ m/s}^2}$

(c) $a_r = \dfrac{v^2}{r}$

$v = \sqrt{a_r r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67 \text{ m/s at } 36.9° \text{ above the horizontal}}$

4.36 (a) $\mathbf{v}_H = 0 + \mathbf{a}_H t = (3.00\mathbf{i} - 2.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})$

$\mathbf{v}_H = (15.0\mathbf{i} - 10.0\mathbf{j}) \text{ m/s}$

$\mathbf{v}_J = 0 + \mathbf{a}_j t = (1.00\mathbf{i} + 3.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})$

$\mathbf{v}_J = (5.00\mathbf{i} + 15.0\mathbf{j}) \text{ m/s}$

$\mathbf{v}_{HJ} = \text{v}_H - \text{v}_J = (15.0\mathbf{i} - 10.0\mathbf{j} - 5.00\mathbf{i} - 15.0\mathbf{j}) \text{ m/s}$

$\mathbf{v}_{HJ} = (10.0\mathbf{i} - 25.0\mathbf{j}) \text{ m/s}$

$|\mathbf{v}_{HJ}| = \sqrt{(10.0)^2 + (25.0)^2} \text{ m/s} = \boxed{26.9 \text{ m/s}}$

(b) $\mathbf{r}_H = 0 + 0 + \dfrac{1}{2}\mathbf{a}_H t^2 = \dfrac{1}{2}(3.00\mathbf{i} - 2.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})^2$

$\mathbf{r}_H = (37.5\mathbf{i} - 25.0\mathbf{j}) \text{ m}$

$\mathbf{r}_J = \dfrac{1}{2}(1.00\mathbf{i} + 3.00\mathbf{j}) \text{ m/s}^2 (5.00 \text{ s})^2 = (12.5\mathbf{i} - 37.5\mathbf{j}) \text{ m}$

$\mathbf{r}_{HJ} = \mathbf{r}_H - \mathbf{r}_J = (37.5\mathbf{i} - 25.0\mathbf{j} - 12.5\mathbf{i} - 37.5\mathbf{j}) \text{ m}$

$\mathbf{r}_{HJ} = (25.0\mathbf{i} - 62.5\mathbf{j}) \text{ m}$

$|\mathbf{r}_{HJ}| = \sqrt{(25.0)^2 + (62.5)^2} \text{ m} = \boxed{67.3 \text{ m}}$

(c) $\mathbf{a}_{HJ} = \mathbf{a}_H - \mathbf{a}_J = (3.00\mathbf{i} - 2.00\mathbf{j} - 1.00\mathbf{i} - 3.00\mathbf{j}) \text{ m/s}^2$

$\mathbf{a}_{HJ} = \boxed{(2.00\mathbf{i} - 5.00\mathbf{j}) \text{ m/s}^2}$

4.37 Total time in still water $t = \dfrac{d}{v} = \dfrac{2000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$

Total time = time upstream plus time downstream

$$t_{\text{up}} = \frac{1000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$$

$$t_{\text{down}} = \frac{1000}{(1.20 + 0.500)} = 588 \text{ s}$$

$$t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}$$

Goal Solution

G: If we think about the time for the trip as a function of the stream's speed, we realize that if the stream is flowing at the same rate or faster than the student can swim, he will never reach the 1.00 km mark even after an infinite amount of time. Since the student can swim 1.20 km in 1000 s, we should expect that the trip will definitely take longer than in still water, maybe about 2000 s (~30 minutes).

O: The total time in the river is the longer time upstream (against the current) plus the shorter time downstream (with the current). For each part, we will use the basic equation $t = d/v$, where v is the speed of the student relative to the shore.

A: $$t_{\text{up}} = \frac{d}{v_{\text{student}} - v_{\text{stream}}} = \frac{1000 \text{ m}}{1.20 \text{ m/s} - 0.500 \text{ m/s}} = 1429 \text{ s}$$

$$t_{\text{dn}} = \frac{d}{v_{\text{student}} + v_{\text{stream}}} = \frac{1000 \text{ m}}{1.20 \text{ m/s} + 0.500 \text{ m/s}} = 588 \text{ s}$$

Total time in river, $t_{\text{river}} = t_{\text{up}} + t_{\text{dn}} = 2.02 \times 10^3$ s

In still water, $t_{\text{still}} = \dfrac{d}{v} = \dfrac{2000 \text{ m}}{1.20 \text{ m/s}} = 1.67 \times 10^3$ s therefore, $t_R = 1.21 t_{\text{still}}$

L: As we predicted, it does take the student longer to swim up and back in the moving stream than in still water (21% longer in this case), and the amount of time agrees with our estimation.

4.38 The bumpers are initially 100 m = 0.100 km apart. After time t the bumper of the leading car travels $40.0t$, while the bumper of the chasing car travels $60.0t$.

Since the cars are bumper-to-bumper at time t, we have

$$0.100 + 40.0t = 60.0t, \text{ yielding } t = 5.00 \times 10^{-3} \text{ h} = \boxed{18.0 \text{ s}}$$

4.39 $v = (150^2 + 30.0^2)^{1/2} = \boxed{153\text{ km/h}}$

$$\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = \boxed{11.3°} \quad \text{north of west}$$

4.40 For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$. Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1 - v^2/c^2}}$$

For Beth, her cross-stream speed (both ways) is $\sqrt{c^2 - v^2}$

Thus, the total time for Beth is

$$t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{2L/c}{\sqrt{1 - v^2/c^2}}}$$

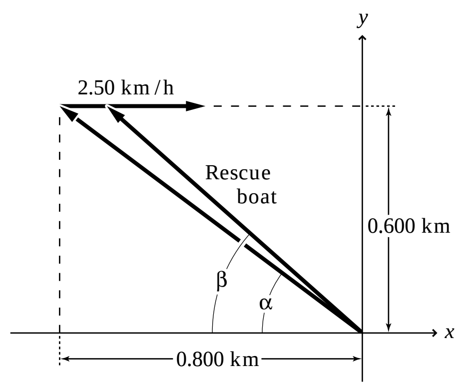
Since $1 - \dfrac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

4.41 α = Heading with respect to the shore

β = Angle of boat with respect to the shore

(a) The boat should always steer for the child at heading

$$\alpha = \tan^{-1}\frac{0.600}{0.800} = \boxed{36.9°}$$



(b) $v_x = 20.0\cos\alpha - 2.50 = 13.5\text{ km/h}$

$v_y = 20.0\sin\alpha = 12.0\text{ km/h}$

$$\beta = \tan^{-1}\left(\frac{12.0\text{ km/h}}{13.5\text{ km/h}}\right) = \boxed{41.6°}$$

(c) $t = \dfrac{d_y}{v_y} = \dfrac{0.600\text{ km}}{12.0\text{ km/h}} = 5.00 \times 10^{-2}\text{ h} = \boxed{3.00\text{ min}}$

4.42 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

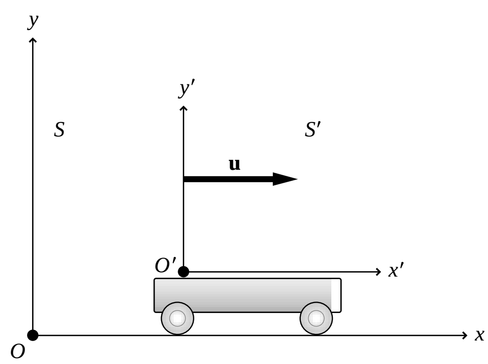
$$a = \sqrt{(2.50\ \text{m/s})^2 + (9.80\ \text{m/s})^2} = \boxed{10.1\ \text{m/s}^2}$$

$$\tan\theta = \frac{2.50\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.255$$

$$\theta = \boxed{14.3°\ \text{to the south from the vertical}}$$

(b) $a = \boxed{9.80\ \text{m/s}^2\ \text{vertically downward}}$

4.43 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have



a

$$\frac{v_y'}{v_x'} = \tan 60.0° = \sqrt{3}$$

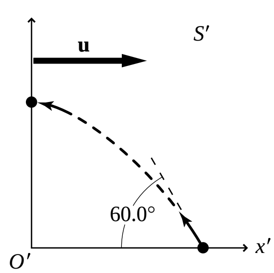
Then, because there is no x-motion in S, we can write $v_x = v_y' + u = 0$ so that $v_x' = -u = -10.0\ \text{m/s}$. Hence the ball is thrown backwards in S'. Then,

$$v_y = v_y' = \sqrt{3}\left|v_x'\right| = 10.0\sqrt{3}\ \text{m/s}$$

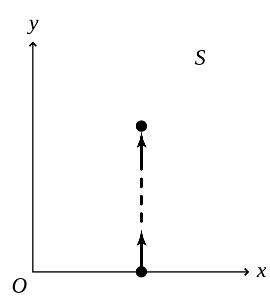
Using $v_y^2 = 2gh$ (from Eq. 4.13), we find

$$h = \frac{(10.0\sqrt{3}\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{15.3\ \text{m}}$$

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).



b



c

4.44 Equation 4.13: $h = \dfrac{v_i^2 \sin^2 \theta_i}{2g}$

Equation 4.14: $R = \dfrac{v_i^2 \sin 2\theta_i}{g} = \dfrac{2v_i^2 \sin \theta_i \cos \theta_i}{g}$

If $h = R/6$, Equation 4.13 yields [$v_i \sin \theta_i = \sqrt{gR/3}$] (1)

Substituting the result given in Equation (1) above into Equation 4.14 gives

$$R = \frac{2(\sqrt{gR/3})v_i \cos \theta_i}{g}$$

which reduces to [$v_i \cos \theta_i = \frac{1}{2}\sqrt{3gR}$] (2)

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path (where $v_{yf} = 0$) is found to be $t_{\text{peak}} = \dfrac{v_i \sin \theta_i}{g}$. Using Equation (1), this gives $t_{\text{peak}} = \sqrt{\dfrac{R}{3g}}$. The total time of the ball's flight is then

$$t_{\text{flight}} = 2t_{\text{peak}} = \boxed{2\sqrt{\frac{R}{3g}}}$$

(b) At the peak of the path, the ball moves horizontally with speed

$$v_{\text{peak}} = v_{xi} = v_i \cos \theta_i$$

Using Equation (1), this becomes $v_{\text{peak}} = \boxed{\frac{1}{2}\sqrt{3gR}}$.

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin \theta_i$ and, from Equation (1), this is

$$v_{yi} = \boxed{\sqrt{gR/3}}$$

(d) Squaring Equations (1) and (2) and adding the results gives

$$v_i^2(\sin^2 \theta_i + \cos^2 \theta_i) = \frac{gR}{3} + \frac{3gR}{4} = \frac{13gR}{12}$$

Thus, the initial speed is $v_i = \boxed{\sqrt{\dfrac{13gR}{12}}}$.

(e) Dividing Equation (1) by (2) yields

$$\tan\theta_i = \frac{v_i\sin\theta_i}{v_i\cos\theta_i} = \left[\frac{\left(\sqrt{gR/3}\right)}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \frac{2}{3}$$

Therefore, $\theta_i = \tan^{-1}\left(\frac{2}{3}\right) = \boxed{33.7°}$.

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0°$. With the speed found in (d), Equation 4.13 then yields

$$h_{\max} = \frac{(13\,gR/12)\sin^2 90.0°}{2g} = \boxed{\frac{13R}{24}}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0°$. With the speed found in (d), Equation 4.14 then gives

$$R_{\max} = \frac{(13gR/12)\sin 90.0°}{g} = \boxed{\frac{13R}{12}}$$

4.45 At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2\,|\,x(t)\,| = 2(v_{xi}t) = 2(v_i\cos\theta_i)t = \boxed{2v_it\cos\theta_i}$$

4.46 After the string breaks the ball is a projectile, for time t in

$$y = v_{yi}t + \frac{1}{2}\,a_y\,t^2$$

$$-1.20\text{ m} = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)\,t^2$$

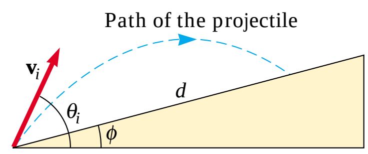
$$t = 0.495\text{ s}$$

Its constant horizontal speed is

$$v_x = \frac{x}{t} = \frac{2.00\text{ m}}{0.495\text{ s}} = 4.04\text{ m/s}$$

so before the string breaks

$$a_c = \frac{v^2}{r} = \frac{(4.04\text{ m/s})^2}{0.300\text{ m}} = \boxed{54.4\text{ m/s}^2}$$



4.47 (a) $y = \tan(\theta_i)\, x - \dfrac{g}{2v_i^2 \cos^2(\theta_i)}\, x^2$

Setting $x = d\cos(\phi)$, and $y = d\sin(\phi)$, we have

$$d\sin(\phi) = \tan(\theta_i) d\cos(\phi) - \frac{g}{2v_i^2\cos^2(\theta_i)}(d\cos(\phi))^2$$

Solving for d yields,

$$d = \frac{2v_i^2 \cos(\theta_i)\,[\sin(\theta_i)\cos(\phi) - \sin(\phi)\cos(\theta_i)]}{g\cos^2(\phi)}$$

or $\quad \boxed{d = \dfrac{2v_i^2\cos(\theta_i)\sin(\theta_i - \phi)}{g\cos^2(\phi)}}$

(b) Setting $\dfrac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \dfrac{\phi}{2}}$ and

$$\boxed{d_{max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$$

4.48 (a)(b) Since the shot leaves the gun horizontally, the time it takes to reach the target is $t = \dfrac{x}{v_i}$.

The vertical distance traveled in this time is

$$y = -\frac{1}{2}gt^2 = -\frac{g}{2}\left(\frac{x}{v_i}\right)^2 = Ax^2$$

where $\boxed{A = -\dfrac{g}{2v_i^2}}$

(c) If $x = 3.00$ m, $y = -0.210$ m, then $A = \dfrac{-0.210}{9.00} = -2.33 \times 10^{-2}$

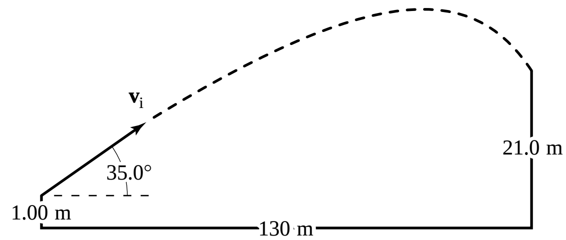
$$v_i = \sqrt{\frac{-g}{2A}} = \sqrt{\frac{-9.80}{-4.66\times 10^{-2}}}\ \text{m/s} = \boxed{14.5\ \text{m/s}}$$

4.49 Refer to the sketch:

(a) & (b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i \cos 35.0°)t$

$\Delta y = v_{yi}t + \frac{1}{2}at^2$ substitution yields

$20.0 = (v_i \sin 35.0°)t + \frac{1}{2}(-9.80)\ t^2$



Solving the above gives $t = \boxed{3.81\ \text{s}}$

$v_i = \boxed{41.7\ \text{m/s}}$

(c) $v_y = v_i \sin \theta_i - gt$

$v_x = v_i \cos \theta_i$

At $t = 3.81$ s, $v_y = 41.7 \sin 35.0° - (9.80)(3.81) = \boxed{-13.4\ \text{m/s}}$

$v_x = (41.7 \cos 35.0°) = \boxed{34.1\ \text{m/s}}$

$v = \sqrt{v_x^2 + v_y^2} = \boxed{36.6\ \text{m/s}}$

4.50 (a) The moon's gravitational acceleration is the bullet's centripetal acceleration:

(For the moon's radius, see endpapers of text.)

$$a = \frac{v^2}{r}$$

$$\left(\frac{1}{6}\right) 9.80\ \text{m/s}^2 = \frac{v^2}{1.74 \times 10^6\ \text{m}}$$

$$v = \sqrt{2.84 \times 10^6\ \text{m}^2/\text{s}^2} = \boxed{1.69\ \text{km/s}}$$

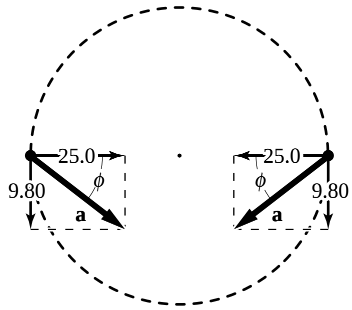
(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6\ \text{m})}{1.69 \times 10^3\ \text{m/s}} = 6.47 \times 10^3\ \text{s} = \boxed{1.80\ \text{h}}$$

4.51 (a) $a_r = \dfrac{v^2}{r} = \dfrac{(5.00\text{ m/s})^2}{1.00\text{ m}} = \boxed{25.0\text{ m/s}^2}$

$a_T = g = \boxed{9.80\text{ m/s}^2}$

(b)



(c) $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(25.0\text{ m/s}^2)^2 + (9.80\text{ m/s})^2} = \boxed{26.8\text{ m/s}^2}$

$\phi = \tan^{-1}\left(\dfrac{a_t}{a_r}\right) = \tan^{-1}\dfrac{9.80\text{ m/s}^2}{25.0\text{ m/s}^2} = \boxed{21.4°}$

4.52 $x = v_{ix}t = v_i t\cos 40.0°$ Thus, when $x = 10.0$ m,

$$t = \frac{10.0\text{ m}}{v_i\cos 40.0°}$$

At this time, y should be 3.05 m – 2.00 m = 1.05 m.

Thus, $1.05\text{ m} = \dfrac{(v_i\sin 40.0°)\,10.0\text{ m}}{v_i\cos 40.0°} + \dfrac{1}{2}(-9.80\text{ m/s}^2)\left[\dfrac{10.0\text{ m}}{v_i\cos 40.0°}\right]^2$

From this, $v_i = \boxed{10.7\text{ m/s}}$

***4.53** At $t = 2.00$ s, $v_x = 4.00$ m/s

$v_y = -8.00$ m/s

$v = \sqrt{v_x^2 + v_y^2} = \boxed{8.94\text{ m/s}}$

$\theta = \tan^{-1}\dfrac{v_y}{v_x} = \boxed{-63.4°\text{, below horizontal}}$

4.54 The special conditions allowing use of Equation 4.14 apply.

For the ball thrown at 45.0°, $D = R_{45} = \dfrac{v_i^2\ 1}{g}$

For the bouncing ball, $D = R_1 + R_2 = \dfrac{v_i^2 \sin 2\theta}{g} + \dfrac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$ where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5} \qquad \boxed{\theta = 26.6°}$$

(b) The time for any symmetric parabolic flight is given by

$$y = v_{yi}t - \frac{1}{2}\ gt^2$$

$$0 = v_i \sin \theta_i t - \frac{1}{2}\ gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \dfrac{2v_i \sin \theta_i}{g}$ is the time at landing.

So, for the ball thrown at 45.0°

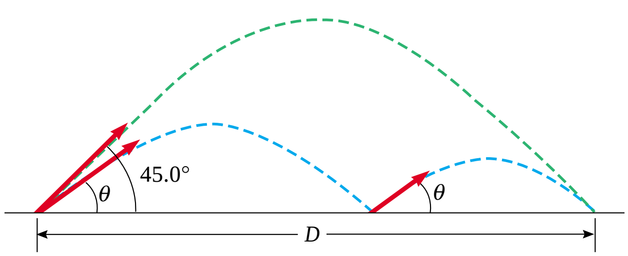
$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/\mathrm{g}}{2v_i \sin 45.0°/\mathrm{g}} = \frac{1.34}{1.41} = \boxed{0.949}$$



4.55 From Equation 4.13, the maximum height a ball can reach is

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

For a throw straight up, $\theta_i = 90°$ and $h = \frac{v_i^2}{2g}$.

From Equation 4.14 the range a ball can be thrown is $R = \frac{v_i^2 \sin 2\theta}{g}$.

For maximum range, $\theta = 45°$ and $R = \frac{v_i^2}{g}$.

Therefore for the same v_i, $h = \frac{R}{2} = \frac{40.0 \text{ m}}{2} = \boxed{20.0 \text{ m}}$

4.56. Using the range equation (Equation 4.14)

$$R = \frac{v_i^2 \sin (2\theta_i)}{g}$$

the maximum range occurs when $\theta_i = 45°$, and has a value $R = \frac{v_i^2}{g}$.

Given R, this yields $v_i = \sqrt{gR}$

If the boy uses the same speed to throw the ball vertically upward, then

$$v_y = \sqrt{gR} - gt \qquad \text{and} \qquad y = \sqrt{gR}\; t - \frac{gt^2}{2}$$

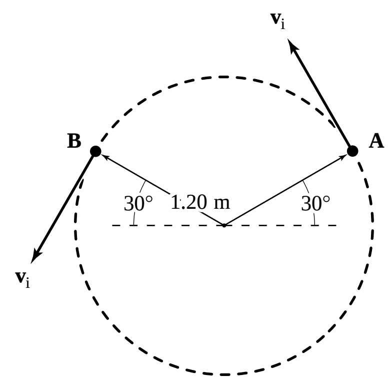
at any time, t.

At the maximum height, $v_y = 0$, giving $t = \sqrt{\frac{R}{g}}$, and so the maximum height reached is

$$y_{\max} = \sqrt{gR}\sqrt{\frac{R}{g}} - \frac{g}{2}\left(\sqrt{\frac{R}{g}}\right)^2 = R - \frac{R}{2} = \boxed{\frac{R}{2}}$$

4.57 Choose upward as the positive y-direction and leftward as the positive x-direction. The vertical height of the stone when released from A or B is

$$y_i = (1.50 + 1.20 \sin 30.0°) \text{ m} = 2.10 \text{ m}$$



(a) The equations of motion after release at A are

$$v_y = v_i \sin 60.0° - gt = (1.30 - 9.80t) \text{ m/s}$$

$$v_x = v_i \cos 60.0° = 0.750 \text{ m/s}$$

$$y = (2.10 + 1.30t - 4.90t^2) \text{ m}$$

$$\Delta x_A = (0.750t) \text{ m}$$

When $y = 0$, $t = \dfrac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800 \text{ s}$

Then, $\Delta x_A = (0.750)(0.800) \text{ m} = \boxed{0.600 \text{ m}}$

(b) The equations of motion after release at point B are

$$v_y = v_i(-\sin 60.0°) - gt = (-1.30 - 9.80t) \text{ m/s}$$

$$v_x = v_i \cos 60.0° = 0.750 \text{ m/s}$$

$$y_i = (2.10 - 1.30t - 4.90t^2) \text{ m}$$

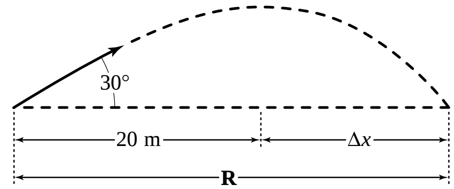
When $y = 0$, $t = \dfrac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536 \text{ s}$

Then, $\Delta x_B = (0.750)(0.536) \text{ m} = \boxed{0.402 \text{ m}}$

(c) $a_r = \dfrac{v^2}{r} = \dfrac{(1.50 \text{ m/s})^2}{1.20 \text{ m}} = \boxed{1.87 \text{ m/s}^2 \text{ toward the center}}$

(d) After release, $\mathbf{a} = -g\mathbf{j} = \boxed{9.80 \text{ m/s}^2 \text{ downward}}$

4.58 The football travels a horizontal distance



$$R = \frac{v_i^2 \sin(2\theta_i)}{g} = \frac{(20.0)^2 \sin(60.0°)}{9.80} = 35.3 \text{ m}$$

Time of flight of ball is

$$t = \frac{2v_i \sin\theta_i}{g} = \frac{2(20.0)\sin 30.0°}{9.80} = 2.04 \text{ s}$$

The receiver is Δx away from where the ball lands and $\Delta x = 35.3 - 20.0 = 15.3$ m.

To cover this distance in 2.04 s, he travels with a velocity

$$v = \frac{15.3}{2.04} = \boxed{7.50 \text{ m/s in the direction the ball was thrown}}$$

4.59 (a) $\Delta y = -\frac{1}{2} gt^2$; $\Delta x = v_i t$. Combine the equations eliminating t:

$$\Delta y = -\frac{1}{2} g\left(\frac{\Delta x}{v_i}\right)^2 \text{ from this } (\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right) v_i^2$$

$$\text{thus } \Delta x = v_i\sqrt{\frac{-2\,\Delta y}{g}} = 275\sqrt{\frac{-2(-300)}{9.80}} = 6.80 \times 10^3 \text{ m} = \boxed{6.80 \text{ km}}$$

(b) The plane has the same velocity as the bomb in the x direction.

Therefore, the plane will be $\boxed{\text{3000 m directly above the bomb}}$ when it hits the ground.

(c) When θ is measured from the vertical, $\tan\theta = \frac{\Delta x}{\Delta y}$; therefore,

$$\theta = \tan^{-1}\frac{\Delta x}{\Delta y} = \tan^{-1}\left(\frac{6800}{3000}\right) = \boxed{66.2°}$$

4.60 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - gx^2/2v_i^2$

(a) The elevation y_r of points on the rock is described by $y_r^2 + x^2 = R^2$. We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

We get the strictest requirement for x approaching zero. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock:

$$1 > \frac{gR}{v_i^2} \qquad \boxed{v_i > \sqrt{gR}}$$

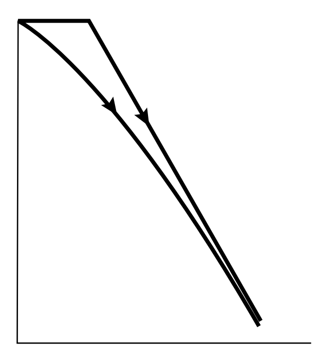
(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$ or $x = \sqrt{2}\,R$

The distance from the rock's base is $x - R = \boxed{(\sqrt{2} - 1)R}$

4.61 (a) From Part (C), the raptor dives for 6.34 – 2.00 = 4.34 s

undergoing displacement 197 m downward and (10.0)(4.34) = 43.4 m forward.

$$v = \frac{\Delta d}{\Delta t} = \frac{\sqrt{(197)^2 + (43.4)^2}}{4.34} = \boxed{46.5\ \text{m/s}}$$



(b) $\alpha = \tan^{-1}\left(\frac{-197}{43.4}\right) = \boxed{-77.6°}$

(c) $197 = \frac{1}{2}gt^2 \qquad \boxed{t = 6.34\ \text{s}}$

Goal Solution

G: We should first recognize that the hawk cannot instantaneously change from slow horizontal motion to rapid downward motion. The hawk cannot move with infinite acceleration. We assume that the time required for the hawk to accelerate is short compared to two seconds. Based on our everyday experiences, a reasonable diving speed for the hawk might be about 100 mph (~ 50 m/s) downwards and should last only a few seconds.

O: We know the distance that the mouse and hawk fall, but to find the diving speed of the hawk, we must know the time of descent. If the hawk and mouse both maintain their original horizontal velocity of 10 m/s (as they should without air resistance), then the hawk only needs to think about diving straight down, but to a ground-based observer, the path will appear to be a straight line angled less than 90° below horizontal.

A: The mouse falls a total vertical distance, $y = 200\text{ m} - 3.00\text{ m} = 197\text{ m}$

The time of fall is found from $y = v_{yi}t - \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2(197\text{ m})}{9.80\text{ m/s}^2}} = 6.34\text{ s}$$

To find the diving speed of the hawk, we must first calculate the total distance covered from the vertical and horizontal components. We already know the vertical distance, y, so we just need the horizontal distance during the same time (minus the 2.00 s late start).

$$x = v_{xi}(t - 2.00\text{ s}) = (10.0\text{ m/s})(6.34\text{ s} - 2.00\text{ s}) = 43.4\text{ m}$$

The total distance is $d = \sqrt{x^2 + y^2} = \sqrt{(43.4\text{ m})^2 + (197\text{ m})^2} = 202\text{ m}$

So the hawk's diving speed is $v_{\text{hawk}} = \dfrac{d}{t - 2.00\text{ s}} = \dfrac{202\text{ m}}{4.34\text{ s}} = 46.5\text{ m/s}$

At an angle of $\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{197\text{ m}}{43.4\text{ m}}\right) = 77.6°$ below the horizontal

L: The answers appear to be consistent with our predictions, even thought it is not possible for the hawk to reach its diving speed in zero time. Sometimes we must make simplifying assumptions to solve complex physics problems, and sometimes these assumptions are not physically possible. Once the idealized problem is understood, we can attempt to analyze the more complex, real-world problem. For this problem, if we considered the realistic effects of air resistance and the maximum diving acceleration attainable by the hawk, we might find that the hawk could not catch the mouse before it hit the ground.

4.62 (1) Equation of bank (2) and (3) are the equations of motion

(1) $y^2 = 16x$ (2) $x = v_i t$ (3) $y = -\frac{1}{2} g t^2$

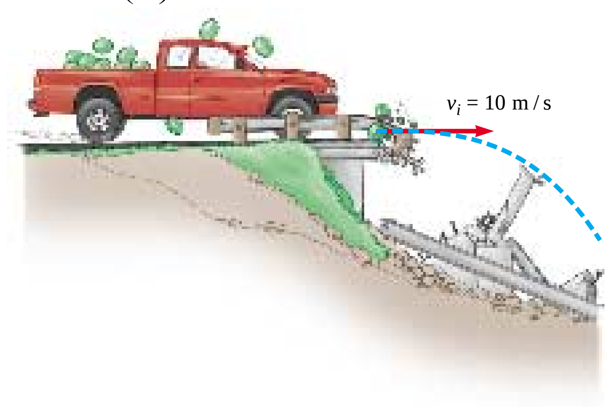
Substitute for t from (2) into (3) $y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)$

Equate y from the bank equation to y from the equations of motion:

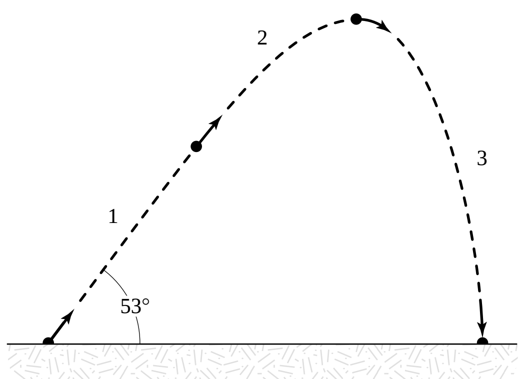
$$16x = \left[-\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2 x^4}{4v_i^4} - 16x = x\left(\frac{g^2 x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64 v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$

Also, $y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right) = \frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}$



4.63 Consider the rocket's trajectory in 3 parts as shown in the sketch.



Our initial conditions give:

$a_y = 30.0 \sin 53.0° = 24.0 \text{ m/s}^2$; $a_x = 30.0 \cos 53.0° = 18.1 \text{ m/s}^2$

$v_{yi} = 100 \sin 53.0° = 79.9 \text{ m/s}$; $v_{xi} = 100 \cos 53.0° = 60.2 \text{ m/s}$

The distances traveled during each phase of the motion are given in the table.

Path #1: $v_{yf} - 79.9 = (24.0)(3.00)$ or $v_{yf} = 152$ m/s

$v_{xf} - 60.2 = (18.1)(3.00)$ or $v_{xf} = 114$ m/s

$$\Delta y = (79.9)(3.00) + \frac{1}{2}(24.0)(3.00)^2 = 347 \text{ m}$$

$$\Delta x = (60.2)(3.00) + \frac{1}{2}(18.1)(3.00)^2 = 262 \text{ m}$$

Path #2: $a_x = 0$, $v_{xf} = v_{xi} = 114$ m/s

$0 - 152 = -(9.80)t$ or $t = 15.5$ s

$\Delta x = (114)(15.5) = 1.77 \times 10^3$ m;

$$\Delta y = (152)(15.5) - \frac{1}{2}(9.80)(15.5)^2$$

$$= 1.17 \times 10^3 \text{ m}$$

Path #3: $(v_{yf})^2 - 0 = 2(-9.80)(-1.52 \times 10^3)$

or $v_{yf} = -173$ m/s

$v_{xf} = v_{xi} = 114$ m/s since $a_x = 0$

$-173 - 0 = -(9.80)t$ or $t = 17.6$ s

$\Delta x = (114)(17.7) = 2.02 \times 10^3$ m

	Path Part		
	#1	#2	#3
a_y	24.0	–9.80	–9.80
a_x	18.1	0.0	0.00
v_{yf}	152	0.0	–173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.00
v_{xi}	60.2	114	114
Δy	347	1.17×10^3	-1.52×10^3
Δx	262	1.77×10^3	2.02×10^3
t	3.00	15.5	17.6

(a) $\Delta y(\text{max}) = \boxed{1.52 \times 10^3 \text{ m}}$

(b) $t(net) = 3.00 + 15.5 + 17.7 = \boxed{36.1 \text{ s}}$

(c) $\Delta x(net) = 262 + 1.77 \times 10^3 + 2.02 \times 10^3 = \boxed{4.05 \times 10^3 \text{ m}}$

4.64 Let V = boat's speed in still water and v = river's speed and let d = distance traveled upstream in $t_1 = 60.0$ min and t_2 = time of return. Then, for the log, $1000 \text{ m} = vt = v(t_1 + t_2)$, and for the boat, $d = (V - v)t_1$; $(d + 1000) = (V + v)t_2$; and $t = t_1 + t_2$

Combining the above gives

$$\frac{1000}{v} = \frac{d}{(V - v)} + \frac{d + 1000}{(V + v)}$$

Substituting for $d = (V - v)(3600)$ gives $v = \boxed{0.139 \text{ m/s}}$

4.65 (a) While on the incline:

$v^2 - v_i^2 = 2a\,\Delta x$ $\qquad$ $v - v_i = at$

$v^2 - 0 = 2(4.00)(50.0)$ $\qquad$ $20.0 - 0 = 4.00t$

$v = \boxed{20.0 \text{ m/s}}$ $\qquad$ $t = \boxed{5.00 \text{ s}}$

(b) Initial free-flight conditions give us

$v_{xi} = 20.0 \cos 37.0° = 16.0$ m/s; $v_{yi} = -20.0 \sin 37.0° = -12.0$ m/s

$v_x = v_{xi}$ since $a_x = 0$;

$v_y = -(2a_y\,\Delta y + v_{yi}^2)^{1/2} = -[2(-9.80)(-30.0) + (-12.0)^2]^{1/2} = -27.1$ m/s

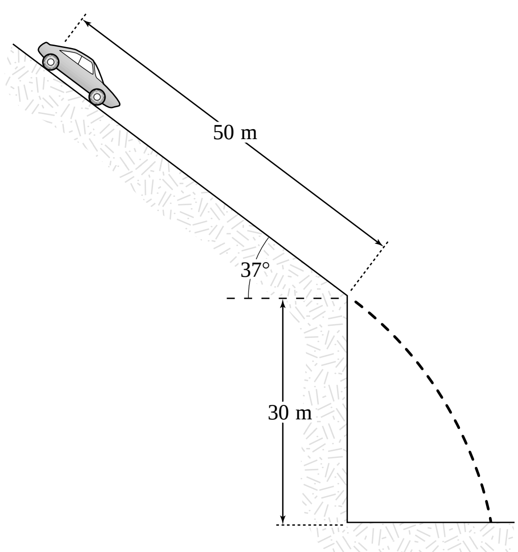
$v = (v_x^2 + v_y^2)^{1/2} = [(16.0)^2 + (-27.1)^2]^{1/2}$

$= \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$

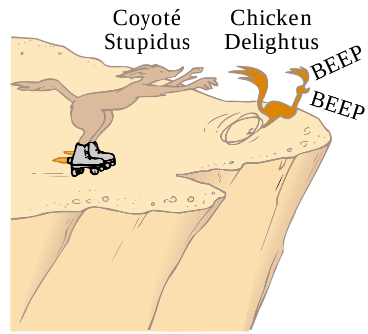
(c) $t_1 = 5$ s; $t_2 = \dfrac{(v_y - v_{yi})}{a_y} = \dfrac{(-27.1 + 12.0)}{-9.80} = 1.54$ s

$t = t_1 + t_2 = \boxed{6.54 \text{ s}}$

(d) $\Delta x = v_{xi}t_1 = (16.0)(1.54) = \boxed{24.6 \text{ m}}$



4.66 (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)\,t^2$



Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get $v_i = \boxed{22.9\text{ m/s}}$ and $t = 3.06$ s

(b) At the edge of the cliff $v_{xi} = at = (15.0)(3.06) = 45.8$ m/s

$$\Delta y = \frac{1}{2}a_y t^2$$

Substituting we find $-100 = \frac{1}{2}(-9.80)\,t^2$

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)t + \frac{1}{2}(15.0)\,t^2$$

Solving the above gives $\Delta x = \boxed{360\text{ m}}$ $t = 4.52$ s

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t$$

$$v_{xf} = 45.8 + 15(4.52)$$

$$v_{xf} = \boxed{114\text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t$$

$$= 0 - 9.80(4.52)$$

$$v_{yf} = \boxed{-44.3\text{ m/s}}$$

4.67 (a) $\Delta x = v_{xi}t$, $\Delta y = v_{yi}t + \frac{1}{2}gt^2$,

$d\cos 50.0° = (10.0\cos 15.0°)t$, and

$d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)\,t^2$

Solving the above gives

$d = \boxed{43.2\text{ m}}$ $t = 2.87$ s

(b) Since $a_x = 0$,

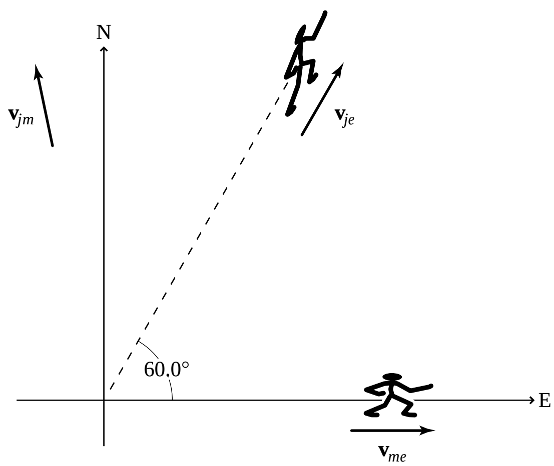
$$v_{xf} = v_{xi} = 10.0 \cos 15.0° = \boxed{9.66 \text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = (10.0 \sin 15.0°) - (9.80)(2.87) = \boxed{-25.5 \text{ m/s}}$$

Air resistance would decrease the values of the range and maximum height.

As an air foil he can get some lift and increase his distance.

4.68 Define **i** to be directed East, and **j** to be directed North.



According to the figure, set

$\mathbf{v}_{je}$ = velocity of Jane, relative to the earth

$\mathbf{v}_{me}$ = velocity of Mary, relative to the earth

$\mathbf{v}_{jm}$ = velocity of Jane, relative to Mary,

Such that $\mathbf{v}_{je} = \mathbf{v}_{jm} + \mathbf{v}_{me}$

Solve for part (b) first. By the figure,

$$\mathbf{v}_{je} = [5.40(\cos 60.0°)\mathbf{i} + 5.40(\sin 60.0°)\mathbf{j}] \text{ m/s}$$

$$= (2.70\mathbf{i} + 4.68\mathbf{j}) \text{ m/s}$$

and $\mathbf{v}_{me} = 4.00\mathbf{i}$ m/s

So, (b) $\mathbf{v}_{jm} = \boxed{(-1.30\mathbf{i} + 4.68\mathbf{j}) \text{ m/s}}$

The distance between the two players increases at a rate of $|\mathbf{v}_{jm}|$:

$$|\mathbf{v}_{jm}| = \sqrt{(1.30)^2 + (4.68)^2} \text{ m/s} = 4.86 \text{ m/s}$$

(a) Therefore, $t = \dfrac{d}{\mathbf{v}_{jm}} = \dfrac{25.0 \text{ m}}{4.86 \text{ m/s}} = \boxed{5.14 \text{ s}}$

(c) After 4 s, $d = \mathbf{v}_{jm} t = (4.86 \text{ m/s})(4.00) = \boxed{19.4 \text{ m apart}}$

***4.69** Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius
60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}} \cong 9\text{ m/s}$$

and its centripetal acceleration is

$$\frac{v^2}{r} \cong \frac{(9\text{ m/s})^2}{0.6\text{ m}} \boxed{\sim 10^2\text{ m/s}^2}$$

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

4.70 Find the highest elevation θ_H that will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest elevation θ_L that will clear the mountain peak; this will yield the maximum range under these conditions if both θ_H *and* θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y = v_{yi}t - \frac{1}{2}\ gt^2 = v_i(\sin\ \theta)t - \frac{1}{2}\ gt^2$$

$$x = v_{xi}t = v_i\,(\cos\ \theta)t$$

Thus $t = \dfrac{x}{v_i\cos\ \theta}$

Substitute into the expression for y

$$y = v_i\,(\sin\ \theta)\frac{x}{v_i\cos\ \theta} - \frac{1}{2}\ g\,\textbf{Error!}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ thus $y = x\tan\ \theta - \dfrac{gx^2}{2v_i^2}(\tan^2\theta + 1)$ and

$$0 = \frac{gx^2}{2v_i^2}\ \tan^2\theta - x\tan\ \theta + \frac{gx^2}{2v_i^2} + y$$

Substitute values, use the quadratic formula and find

$\tan\ \theta = 3.905$ or 1.197 which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$

Range (at θ_H) $= \dfrac{v_i^2 \sin 2\theta_H}{g} = 3.07 \times 10^3$ m from enemy ship

$3.07 \times 10^3 - 2500 - 300 = 270$ m from shore

Range (at θ_L) $= \dfrac{v_i^2 \sin 2\theta_L}{g} = 6.28 \times 10^3$ m from enemy ship

$6.28 \times 10^3 - 2500 - 300 = 3.48 \times 10^3$ m from shore

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.