Introduction to Digital Signal Processing

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LCAV - EPFL
- **Digital**
  - Brings experimental data & abstract models together
  - Makes math very simple i.e. *implementable*

- **Signal**
  - Measurement of a varying quantity
  - Experimental data (physics, electronics, astronomy, etc.)

- **Processing**
  - Manipulation of the information content
  - Abstract model (math, computer science, etc.)
A Bit of History and Philosophy

Egypt, 2500 BC:
Egypt, 2500 BC: the Palermo stone.
A Bit of History and Philosophy

USA, 2005 AD: the Dow-Jones Industrial Average
What do these measurements have in common?

- Life-changing phenomena
- Unpredictable patterns
- Discrete set of observations

= Digital Signal Processing

Is a discrete set of measurement a sufficient representation?
Can we formalize this concept?
The Platonic schizophrenia of Western thought.

- Dichotomy between the ideal and the real
- Zeno’s paradoxes
- An odd synergy: calculus and ballistics
Calculus: a lofty ideal at the service of war.

\[ \ddot{x}(t) = \ddot{v}_0 t + (1/2)g t^2 \]

Galileo, 1638
How does an ideal signal look like? Tuning fork:

It’s a function of a real variable!

\[ f(t) = A \sin(2\pi \omega t + \phi) \]

As such, 3 parameters completely describe the signal.
Ideal Signals vs. Real Signals

Tuning forks are boring; Bach is not:

Unfortunately (or fortunately):

\[ f(t) =? \]

How do we deal with real-world signals?
Sampling: we measure the signal value at regular intervals.

\[ x[n] = f(nT_s) \]

Can we do this or are we in one of Zeno’s paradoxes? Yes, we can if the signal is “slow enough”.

Ideal Signals vs. Real Signals
The Sampling Theorem (Nyquist 1920).
Under appropriate “slowness” conditions for $f(t)$ we have:

$$
    f(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\pi(t - nT_s)/T_s)}{\pi(t - nT_s)/T_s}
$$

In a way, the sampling theorem solves one of Zeno’s paradoxes: the infinite and the finite have been reconciled.

The sampling theorem is the "revolving door" into the digital world. We will therefore operate in the digital world only.
The Digital Revolution

Digital signals make our life simpler:

- **Processing:**
  - Sequence of numbers: ideal for computations
  - Development easy (general-purpose hardware)

- **Storage:**
  - Storage is basically media-independent
  - Perfect duplication
  - Digital compression is miraculous

- **Communications:**
  - Transmission schemes independent of data
  - Error correction techniques make it noise-free
Computing the average value of a signal.
Computing the average value of a signal.

\[ \bar{x} = \frac{1}{b - a} \int_{a}^{b} f(t) \, dt \]
Computing the average value of a \textit{digital} signal.
Computing the average value of a *digital* signal.

\[ \bar{x} = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \]
Computing (vertical) speed the “Platonic” way.

\[ x(t) = v_0 t - \frac{1}{2} gt^2 \]
\[ v(t) = \dot{x}(t) = v_0 - gt \]
Computing speed the DSP way.

\[ x[n] \]
Computing speed the DSP way.

\[ v[n] = \frac{(x[n] - x[n - 1])}{T_s} \]
The "Speed Filter":

Position → Processing → Speed
Inside the "Speed Filter":

\[ x[n] \xrightarrow{z^{-1}} x[n-1] \xrightarrow{+} \frac{1}{T_s} \xrightarrow{} v[n] \]

This is a general result: filters' building blocks are just delays, multiplications and additions.
The Digital Revolution: Storage

How do you store a signal?

- In the (not so) old days:
  - Build a physical system (wax cylinders, magnetic tapes, vinyl...)
  - Fragile, data dependent

- Nowadays:
  - Quantize the signal values into binary digits
  - Store in any digital memory support
  - Perfect copies

Signal to noise ratio for digital signals:

\[
\text{SNR} \approx 6 \text{ dB / bit}
\]
How do you deal with large amounts of data? Compression!

<table>
<thead>
<tr>
<th>Signal Type</th>
<th>Default Rate</th>
<th>Compressed Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Music</strong></td>
<td>4.32 Mbps</td>
<td>128 Kbps MP3</td>
</tr>
<tr>
<td></td>
<td>CD audio</td>
<td></td>
</tr>
<tr>
<td><strong>Voice</strong></td>
<td>64 Kbps</td>
<td>4.8 Kbps CELP</td>
</tr>
<tr>
<td></td>
<td>AM radio</td>
<td></td>
</tr>
<tr>
<td><strong>Image</strong></td>
<td>20 Mb this image</td>
<td>600 Kb JPEG</td>
</tr>
<tr>
<td><strong>Video</strong></td>
<td>170 Mbs PAL video</td>
<td>600-800 Kbs DiVx</td>
</tr>
</tbody>
</table>
The Digital Revolution: Transmission

The Agamemnon, 1858
Digital data allows for large throughputs:

- **Transoceanic cable:**
  - 1866: 8 words per minute ($\approx$5 bps)
  - 1956: AT&T, coax, 48 voice channels ($\approx$3Mbps)
  - 2005: Alcatel Tera10, fiber, 8.4 Tbps ($10^{12}$ bps)
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- **Voiceband modems:**
  - 1950s: Bell 202, 1200 bps
  - 1990s: V90, 56000bps
DSP Friends and Partners

- Electronics
- Computer science
- Physiology
- Music
- Medicine
- Photography
- And many more...
Digital signal processing is FUN!

It’s a fresh new take on what you already studied in theory.

Just turn on a computer and you have a “mad scientist lab” where you can bring everything you know, and nothing ever blows up.